Airline pricing and revenue management

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A simple example

In a 200 seats aircraft, there are empty seats and you are trying to maximize the revenue of the flight

Solution A :
Price set at 400€

Only 120 seats sold
Revenue = 400x120 = 48 000€

Solution B :
Price set at 250€

200 seats sold, load factor is max
Revenue = 250x200 = 50 000€
Why is it possible?

- Unsatisfied demand (load factor < 100%)
- Perishable good
- Fixed cost already paid
- Marginal cost quite low (cost of additional pax)

Solution B:
Price set at 250€

200 seats sold, load factor is max
Revenue = 250x200 = 50 000€

Simple example cont.

In a 200 seats aircraft, there are empty seats and you are trying to maximize the revenue of the flight

Solution A:
Price set at 400€

Only 120 seats sold
Revenue = 400x120 = 48 000€

Solution C:
Price set at 550 €

Only 100 seats sold
Revenue = 550x100 = 55 000€
Why is it possible?

- Demand is heterogeneous (different preferences)
- People with price-inelastic demand
- Flexibility in pricing
- Maximization of revenue vs Max of load factor…

Solution C:
Price set at 550 €

Only 100 seats sold
Revenue = 550x100 = 55000 €

Simple example

In a 200 seats plane, there are empty seats and you are trying to maximize the revenue of the flight

Solution A:
Price set at 400 €

Only 120 seats sold
Revenue = 400x120 = 48000 €

Other solutions ??
Can’t we do better than that?

In the previous example, one only adjusts the price uniformly to increase the revenue (and the load factor). Note that 120 people were ready to pay 400€ for the flight and that the operating costs are (roughly) the same in the two situations.

This is uniform pricing
(one price for all customers)

Can’t we do better than that?

- If we could use that information to separate the consumers into 3 categories:
  - Group 1: ready to pay 550€ (or more!)
  - Group 2: ready to pay 400€
  - Group 3: ready to pay 250€

- How to fill the plane?
  - 100 customers at 550€ Revenue = 55 000€
  - 120-100=20 customers at 400€ = 8 000€
  - 80 remaining seats at 250€ = 20 000€

  Total = 83 000€
Can’t we do better than that?

- Since 120 people were ready to pay 400€ for the flight, do a market segmentation and use price discrimination in order to maximize profit.

This is price discrimination.

Explaining how to achieve profit maximizing through price discrimination is the purpose of this course.

Nathalie LENOIR, October 2011

Part I: Airline Pricing
Airline Pricing...

...follows general pricing principles

General pricing principles

Basic economic principles, price discrimination
Pricing principles: Outline

- The role of prices
- Simple case: homogeneous goods and uniform price
  - Consumer surplus
- Complex cases: price discrimination and/or product differentiation (heterogeneous goods)
  - Price discrimination
  - Effect on surplus
  - Product differentiation

Prices ? What for ?

- To adjust demand and supply
- Example: Prices varying with time can be used to adjust a moving demand to a rigid (limited) supply
A simple case: homogenous good and unique price

- Consider a good with no variation in its composition nor its quality (homogenous)
- Suppose there is a unique price for this good on the market: The seller cannot discriminate among customers and change the price according to their purchasing power
  - This is the case for most goods, with a labeled price

Demand curve and inverse demand

Demand curve: $D(p)$
- Quantity on the y-axis
- Price on the x-axis

Inverse Demand curve: $P(Q)$
- Price on the y-axis
- Quantity on the x-axis
Price and perfect competition

- Assume that each producer has no influence on the price \( p^* \)
  - He is a « price-taker »
  - True if there are many producers on a market
- The producer chooses his production level \( Q^* \) in order to maximize his profit:
  \[
  \text{Max } \Pi(Q) = p^* \times Q - c(Q)
  \]
  Thus \( Q^* \) is such that: \( c'(Q^*) = p^* \)

- The price on the market, \( p^* \), is equal to the marginal cost of production

Price and perfect competition

- One adjusts the quantity produced \( Q^* \) such that \( c'(Q^*) = P^* \)
Price and monopoly

- Consider the extreme case of a monopoly. The producer chooses its price $p_m(Q)$ and its production $Q_m$ as a function of the demand function

- $D(p)$ is reverse to $p(Q)$

\[
\text{Max } \Pi(Q) = p_m(Q) \times Q - C(Q)
\]

so $Q_m$ is such that $C'(Q_m) = p_m(Q) + p_m'(Q_m) \times Q_m$

- The marginal cost is equal to the marginal revenue

- The price is higher and the quantity produced lower (than under perfect competition).

One can show that: $p^m > p^*$, $Q^m < Q^*$, and $\Pi(Q^m) > \Pi(Q^*)$
Price and imperfect competition

- In a case of limited competition (restricted number of producers), the situation lies between the previous cases:
  - Each producer has some flexibility (limited by other producers) for defining its price
- The price lies between the previous prices.

Definition: Consumers surplus

The "consumers surplus" is the area lying between the price paid and the inverse demand curve. This is a measure of the consumers “welfare.”

The surplus is higher under perfect competition: A firm with market power tries to extract the consumers surplus (or rent).
Complex case: price discrimination and/or product differentiation

- If firms are allowed to discriminate among their customers, there may be different prices for the same good (homogenous good)
  - It is called price discrimination (see definition)
- The aims are:
  - surplus extraction (private sector)
  - redistribution (government social measures)
- There may also be differences in the composition or in the quality of the goods (heterogenous goods), leading to a price difference

Price discrimination: definition

There is price “discrimination” if the differences in the prices paid by two customers are not justified by the costs differences of supplying the service or the good
Price discrimination: illustration

Uniform Price $p$

Different levels of prices

Surplus extraction

By setting different prices for different consumers, the producer may extract some of the consumers surplus.
Part of the money extracted from the surplus by setting higher prices for some consumers can be used to define lower prices for others.

Price discrimination

Same good
Different prices
Different consumers
Price discrimination: examples

Why:
- Are movie tickets cheaper for students?
- Are movie tickets cheaper in the morning?
- Do you pay less at “Happy hours” in bars?
- Is car rental cheaper if booked “in advance”?
- Are museum tickets cheaper for kids?
- Are some hotels cheaper during the week(-end)?
- Is advertising on TV more expensive in prime time?
- Are train tickets cheaper for senior?
- Etc ….

Price discrimination: conditions

- The firm must have a sufficient market power (monopoly or oligopoly)
  - Ability to exert influence on price or quantity sold

- Few trade possibility between customers
  - The good is non resalable between customers

- The consumers preferences must be different
  - Different types of consumers (preference, income)
Three types of price discrimination
(Pigou, 1938)

- **1st degree : Perfect discrimination**
  - Theoretical case where the willingness to pay is perfectly known

- **2nd degree : discrimination using filtering and self-selection.**
  - *Ex:* Customers choice : I prefer to go to the movies in the morning and pay less

- **3rd degree : discrimination using signals on consumers preferences**
  - *Ex:* Discount for students, family, etc.

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Price discrimination: Consequences

- The firms extracts parts of the consumers surplus.
- The global effect on the total welfare is not clear
  - The surplus is extracted
  - Results in different prices allowing people with lower willingness to pay, to obtain the good/service
- Very often, there is a redistribution from the consumers with a low price-elasticity (high income) to the consumers with a high price-elasticity (low income)
  - The surplus variation depends on the quantity produced.
The consumers surplus is lower…

Q unchanged, $S > S'$

Uniform Price $p$

Different levels of prices

...unless the quantity produced is changed

Uniform Price $p$

Different levels of prices
Price discrimination

Product

Price 1
Price 2
Price 3

Market segment 1
Market segment 2
Market segment 3

Product differentiation

Product 1
Product 2
Product 3

Price 1
Price 2
Price 3

Market segment 1
Market segment 2
Market segment 3
Price discrimination versus product differentiation

Very often, the difference in the prices paid by two customers is not justified by the cost differences between the goods, so the product differentiation is just a “trick” to make people accept price discrimination.

Product differentiation

- Vertical differentiation: quality variations
- Horizontal differentiation: differences in the product
- Or both
  - Sometimes hard to distinguish (especially for services)
Product differentiation: Example

- Diet coke versus coca-cola
  - Healthier (?) is more expensive…
- Cat A. car rental vs Cat. B
  - Different car, same service
- Le “beurrier Président” vs la “plaquette de beurre Président”
  - Only package differs
- Takeaway vs on table consumption in restaurants
- First and second class ticket in trains
  - Different seats
- Men vs women at the hairdresser
  - Different product? Different quality? Sexism?
- “Ticket bought today vs ticket bought last week ”
  - Add another dimension: time!

Product differentiation and quality

One shows that the quality provided for people with the lowest quality valuation is lowered: the firm use the lowest quality good to segment the market

“What the company is trying to do is prevent the passengers who can pay the second-class ticket fare from traveling third-class; It harms the poor, not because it wants to hurt them but to frighten the rich.”

(Dupuit 1849)
Price discrimination in practice

- Very popular in transportation
  - Motorway tariffs: the cars pay for the trucks (Political decision)
  - Airline and railway pricing: Price discrimination and revenue management
  - Air traffic control pricing: small planes get subsidies from bigger ones
- Can be criticized when the purpose is consumers surplus extraction without competition on the market
- Can be beneficial when production increases

Silly question of the day:

Major airline do not like to sell tickets for single legs. Why?
Answer:

because they cannot control for the purpose of the trip…
…and therefore for the willingness to pay of the passenger
…which makes it more difficult to price discriminate

Example:

- If a passenger buys a ticket one way on Friday morning at a given price, and the return ticket separately (or returns by train or car)
- We do not know if he intends to spend the weekend $\rightarrow \$
- Or go there for a day $\rightarrow $$$ (business traveller!)
Revenue management: Outline

- Chap 1: Revenue management basics
  - Definition, origins and principles
  - Prices and price discrimination
  - Fare classes management
- Chap 2: Single flight capacity control
  - A little bit of statistics
  - Setting the quotas for two fares
  - Limits of the approach
- Chap 3: Revenue management in practice
  I) Dynamic allocation
  II) Nesting
  III) Network pricing

Repay Management

Chapter 1: basics
Revenue management: Outline

❖ Chap 1: Revenue management basics
  ▪ Definition, origins and principles
  ▪ Prices and price discrimination
  ▪ Fare classes management

Definition, Principles and Origins of “Revenue Management”
“Revenue management” is a method for maximizing the total revenues of an airline.

- The goal is different from “simply” having the highest load factor or even from having the highest possible revenue for each passenger (yield)
- The term “yield management” is improper but originally and currently used
- We do not care about yield, but only about total revenues

It requires sophisticated tools

- Information management systems, forecasting models, optimization algorithms… and others

When or where?

- This tool can be used if:
  - The service provided is perishable (not possible to keep it in stock)
  - Capacity is fixed (in the short term)
  - Demand is flexible (sensitive to price changes)
  - Customers have different preferences
  - Prices are free

- Possible in many industries…
  - Airlines, trains, Car rental
  - Hotels…

- But invented by airlines
Origins of «Revenue Management»

- The "Airline Deregulation Act" in 1978 (USA) states the freedom of competition principle
- Freedom of fares
  - Price discrimination is possible
- New entrants (People Express, Southwest…)
  - Low prices for new leisure travellers
- The airlines in activity develop computer programs managing the information and improving marketing strategies
  - Thanks to computerized reservation systems

CRSs

- In 1960, the first Sabre® computerized reservation system (CRS) is installed
- In 1964, it became the largest, private real-time data processing system — second only to the U.S. government’s system. It became an integral part of American Airlines, saving 30 percent on its investments in staff alone.
- By 1978, the Sabre system could store 1 million fares.
First results of deregulation

- Development of new traffic by new airlines
  - Surprise: people ARE actually price-sensitive
  - Lower prices → many new leisure passengers (families, couple, students…)

- Reaction of « old » airlines
  - Concentrate on business customers (but this leaves empty seats!)
  - Or invent a strategy to recapture leisure travellers?

Why not do both?

Basic idea of Revenue Management

- Majors want to keep business travellers
  - Attracted by convenient and frequent schedules
  - Not very sensitive to prices

- And attract leisure travellers by selling them cheap « surplus seats » otherwise empty
  - Since marginal cost is zero once flight is scheduled

- Difficulties to solve
  - Identify correctly the surplus seats: do not sell a seat at cheap price if it could be sold at high price
  - Prevent business customers from buying low priced product
Implementation and first results:

- In 1978 AA implements « super saver fares »
  - Problems to identify « surplus seats ».
  - First a fixed proportion of seats in each flight
  - But flights are all different!
- Full implementation of system of inventory control in 1985 (DINAMO)
  - Identify correctly « surplus » capacity on each flight
  - First revenue management system
⇒ With, as a the direct result, the bankruptcy of People Express in 1986!

Some “facts”

- 1985: DINAMO set up by American Airlines
- 1994: Sabre and SNCF install the RESARAIL™ for the TGV high speed train network (extended to the English Channel Tunnel).
  - Beginning of RM in trains
- American Airlines (pioneer in yield management systems), estimated that RM increased its revenue by $1.4 billion between 1989 and 1991.
- Revenue management increases revenues up to 4-5% !? (Taluri, Van Ryzin)
Revenue management process

Demand forecast
Optimal allocations
Controls!

Important data (history)

pricing and/or initial allocations

Important data (Sales)

Source: Talluri Van Ryzin

Revenue management process cont.

Data Collection
Review Outcomes
Data Analysis
Demand Forecast

Business Forecast
Conversion
Evaluation & Selection

Ops
Admin & Up-sell
Service Delivery
Client Retention

Revenue Management

Generating
Sales

Client Development
Sales Research

Nathalie LENOIR, October 2011
Figures in a major airline

A major: 240 planes; 220 destinations
- 90 flight analysts (= 35 000 (real) O-D)
- 30 pricers (35 000 x 2 ways x 25 fares)
- AMO: technical staff
- Revenue integrity (tools sold by sabre e.g)
- ...
= 220 people (pricing + RM)

Tools

Figure 2: Example of an ad hoc query of bookings by airline and yield class

With Sales Analyzer ad hoc query capabilities, you can create query parameters using any combination of these dimensions: point of sale, airport/pair, carrier, yield class of service, GDS and date range.

Source: Sabre WiseVision
### Fare Distribution for a Sample Market (Seattle - London)

**Source**: Bill Swan (Boeing)

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**Figure 3**: Standard Report showing activity of top agencies in a user-specified market.
Prices and price discrimination

Main Questions:

- **How to set the prices?**
  - Knowledge of demand
  - Comparison with other airlines (market monitoring)
  - Costs

- **How to discriminate between consumers?**
  - By using restrictions on the service provided
  - By using consumer known characteristics (age, status)

- **How to set the capacity of each class?**
  - Accurate demand forecast within each class of price
Prices before and after US deregulation

- **Before US deregulation**
  - Prices fixed by the regulator; two classes (economic and first)
  - Prices linked to distance and not to cost
  
  \[ P_{[A,B]} = \alpha + \beta \times \text{distance } [A,B] \]

- **After**
  - Prices disconnected from distance or cost

Profit maximization before and after deregulation

- **Before:**
  - Competition through frequencies and service to stimulate demand
  - Prices and capacities within “class” quite rigid

- **Strategy:**
  - commit to a capacity depending on competition
  - Price is fixed
  - Maximize load factor
Profit maximization before and after deregulation

- **After**
  - Competition through prices and restrictions
  - Adjustment possible easily
  - Many different fares

- **Strategy**
  - commit to a capacity depending on competition
  - Prices are free
  - Maximize revenues

---

Figures in a major airline

- **Price (for the same seat) varies on a scale of**
  - 1-10 (in economy), 1-20 all services.
  - Example *(Le Monde 14.02.07)*
    - Paris-New-York: 17 fares: 467 € - 3228 € (eco) 8736 € (first class)

- **Revenue**
  - Business = 45% pax; 70% revenue (on medium haul)
  - Business = 15% pax; 45% revenue (on long haul)
Prices!

Prices between Boston and Chicago
http://www.faredetective.com/farehistory/

Prices: current situation

- Prices are adjusted following:
  - Competition (oligopolies!)
  - Passengers characteristics or preferences (willingness to pay): “demand-based pricing”
Prices: current situation

- Prices are adjusted following:
  - Competition (oligopolies!)
  - Passengers characteristics or preferences (willingness to pay): “demand-based pricing”

But...
- Prices are disconnected from costs
  - Prices are defined by strategic considerations
  - The marginal cost is “fuzzy” (close to zero)
- Can airlines completely ignore the cost constraints?
  - Yes in the short term, no in the long run
Costs and Prices

- **Long term decisions/strategies**
  - Choice of fleet: aircraft capacity, range

- **Medium term**
  - Routes, schedules

- **Short term**
  - Pricing does not depend on cost!

- **Results: profits or not?**
  - Review past strategies and improve them

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Restrictions:
the "packages" price-ticket

Airlines propose "menus" or packages with prices and services characteristics

- Numerous price classes: Y, F, J, S, B, M, Q... corresponding to prices
- Characteristics: Origin-destination, but also services and restrictions (date restrictions, no date change, week-end included)
Restrictions examples

- **Third degree discrimination** (objective characteristics):
  - Student prices, family prices, retired people discount

- **Second degree discrimination**
  - Week-end special fares, non-refundable tickets, no date change, special tariff if ticket bought X-days in advance (30, 14, 7)…
  - Goal: discriminate among users considering their willingness to pay, and/or their constraints (time, schedule)

How to set prices?

- The *trade-off* between price and restrictions has to be well studied
  - Good knowledge of demand necessary
  - And learn from past mistakes?

- **Competition outlook**
  - Competition limits the airline power over the consumers

- **Rules of separability, flexibility, and readability**
Pricing rules

- **Separability**
  - Services and prices have to be different enough, but still close enough so that some consumers may accept higher price if lower price not available

- **Flexibility**
  - Ability for the airline to change fares

- **Readability**
  - The tariff has to be clear for consumers

Price and discrimination

- **The different services sold distinguish through prices and quality**
  - The restrictions imposed are variation (worsening) of the service quality

- **Airlines discriminate their consumers using quality and not quantity**
  - It is really discrimination since the variation in quality has a cost quite small for the airline, compared to the variation of the price (price ratio 1 to 10 or more)
Silly question of the day:

Can firms do anything to discriminate?

Silly question of the day: NO

- Pricing discrimination MUST NOT be based on:
  - Sex
  - Race
  - Religion…
- What is legal:
  - Age
  - Occupation
    - In some cases
      - unemployed, retired
  - Quantity sold
Your discount is your age!

Legal, but what is the goal?

Goal: attract older people
- Lower prices only on frames (not on lenses!)
- Profit made on lenses!

Not a real discrimination but rather a trick to attract a certain category of customers (older people wear glasses more than young people, and have more money on average!)
- Marketing!
Your price is your weight!

• Inefficient: what is the point?
• Barely legal (?)
• Not acceptable by society (laws against racial discrimination, sexism, ...)

What do you think of this?

Emma’s Learning Centre

F.6 Use Of English Regular Course (Advanced)

Tutor: Miss Tam
Time: All Thursdays 7:15 - 8:45pm
Fee: $480 *(4 lessons)

* If you got the following grade in the CE English Exam, you can enjoy the following discount.

A grade : 40% off
B grade : 30% off
C grade : 20% off
“Case Study”: your turn!

- Consider using revenue management:
  - In a restaurant (group A)
    - “Pizza type” or “gastronomic”
  - In a hotel (group B)
    - Airport “Hotel” or “Hotel de la plage”
  - In a movie theater (group C)
  - In a Zoo (group D)
  - In a pool (group E)

Is it relevant?

This tool can be used if:
- The service provided is perishable (not possible to keep it in stock)
- Capacity is fixed (in the short term)
- Demand is flexible (sensitive to price changes)
- Customers have different preferences
- Prices are free (by hypothesis)
Questions

- How do you achieve price discrimination?
- Which type (second or third class)?
- Define the “product” you are selling
- Define the fares (and conditions)
- Propose several (new?) price discrimination schemes in this sector
  - Consider management problems and management tools to achieve RM

Questions

- Why is RM possible?
- How to do discrimination
  - Define the various “packages” you are selling
  - Define the new pricing scheme
  - Define the new prices
- What revenue?
  - Conditions for additional expected profit
  - Conditions for no additional expected costs
- Problems?
Solutions

✿ Restaurant:
- Duration of the meal \( \times \) confort (noise, view, hard seats) \( \times \) service (wi-fi, newspapers, TV, …)
- + Classics: (peak-off peak, reservation, location)
- Some extras: (wine by the glass, car wash, etc.)

✿ Hotel
- Duration (good=room/hour) \( \times \) comfort (coins for shower, heater, TV, towels, …) \( \times \) service (wi-fi, newspapers, …)
- + Classics: (peak-off peak, reservation, location)
- Extras: cleaning could be optional

✿ Movie-theater:
- Duration of the movie \( \times \) comfort (choice of seat, sound (headset), 3-D, hard seats) \( \times \) service (wi-fi, newspapers, TV, …)
- + Classics: (peak-off peak, reservation, location)
- Some extras: no queue, sodas, couple seats, subtitles

✿ Pool
- Duration \( \times \) comfort (coins for shower, heater, lockers, towels, …) \( \times \) service (towels…)
- + Classics: peak-off peak, reservation,
- Extras: access to grass, diving area, restricted swimming line, temperature…
Solutions

- **Zoo**
  - Different tours (lions, seals, bears...), duration of the visit (if you have real capacity limits)
  - + Classics: (peak-off peak, reservation, family prices)
  - Some extras: no queue, feeding time, extra for the insectarium...

- **Difficulties: management of capacity in all cases!**
  - Especially if capacity constraints are strong
  - Core of all RM systems
  - If price discrimination leads you to sell all tickets at lower price: YOU LOOSE!

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**Fare classes management: the mechanics of “Revenue Management”**
Fare class management

- **After setting prices and restrictions**
  - 1978 AA « super saver fares »: non refundable, seven days stay, bought 30 days in advance

- **You need to set quotas in each fare class:**
  - Total capacity is fixed (aircraft capacity)
  - In order to sell the right number of « super saver fares », 30 days in advance!
  - To keep seat for business travellers reserving a few days ahead!

- **Already difficult if you deal with only one flight**
  - In real life, airlines deal with a network of connecting flights and passengers

---

An overview of the problems

- **Demand**
  - Is random for each population (uncertainty!)
  - Changes over time
  - Each population has a different pattern over time

- **Airlines**
  - Face a fixed total capacity for each flight
  - Offer different fares (20 or more) for each flight
  - Must allocate seats dynamically
  - Manage multiple flights (network)
Revenue Management

Chapter 2: Single flight capacity control

Revenue management: Outline

- Chap 1: Revenue management basics
  - Definition, origins and principles
  - Prices and price discrimination
  - Fare classes management
- Chap 2: Single flight capacity control
  - A little bit of statistics
  - Setting the quotas for two fares
  - Limits of the approach
- Chap 3: Revenue management in practice
  1) Dynamic allocation
  2) Nesting
  3) Network pricing
All you need is a little bit of Statistics..

You observe the history of demand for a flight/day

Imagine this is the 10am flight every Tuesday, going from Toulouse to Paris...

We trace the history of seats sold over a certain period (92 following weeks)
Silly question of the day:

How to visualize demand?

Answer 1:

Summary statistics

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<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seats</td>
<td>92</td>
<td>41.63</td>
<td>20.2</td>
<td>8</td>
<td>100</td>
</tr>
</tbody>
</table>

\[
\mu = \frac{1}{N} \sum_{i=1}^{N} x_i, \quad \sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2}
\]

Capacity!
Answer 2: Histogram

Answer 3: Box and Whiskers

25th percentile or lower quartile

Median

75th percentile or upper quartile
Answer 4: density

Answer 5: estimation and density modeling
Distribution of demand: density

The demand for a flight is stochastic
q = number of seats sold

Density of demand = \( f(q) \)

What’s important for RM: percentiles!

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<thead>
<tr>
<th>Percentiles</th>
<th>Smallest</th>
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<td>1%</td>
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<tr>
<td>5%</td>
<td>12</td>
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<tr>
<td>10%</td>
<td>18</td>
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<tr>
<td>25%</td>
<td>26</td>
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<td>90%</td>
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<tr>
<td>95%</td>
<td>86</td>
</tr>
<tr>
<td>99%</td>
<td>100</td>
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<tbody>
<tr>
<td>Obs</td>
<td>92</td>
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<tr>
<td>Sum of Wgt.</td>
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<tr>
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<tr>
<td>Kurtosis</td>
<td>3.542707</td>
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</table>

There are 10% of the observations lower than 18
What’s important for RM?

Back on our feet?

Inverse demand curve

Price

Quantity

PH

PM

PL
Back on our feet?

Mean Inverse demand curve

PH
PL
PM

Back on our feet?

Inverse demand curve

PH
PL
PM
The demand for a flight is stochastic
\( q = \text{number of seats sold} \)

Density of demand = \( f(q) \)

\[ E(R_H) = \int_0^\infty P_H \times q \times f_H(q) \, dq \]

with
\[ f_H(q) = \text{prob. of selling exactly } q \text{ seats} \]
Distribution of demand

Probability of having less than 30 “H-seats” sold

\[ \Pr[Q_H \leq 30] = \int_{q=0}^{30} f(q_H) \, dq_H = 15\% = F(30) \]

Cumulative function

\[ F(Q_H) = \Pr[q_H \leq Q_H] = \int_{0}^{Q_H} f(q_H) \, dq_H \]
Some “classics” in statistics

Let $X$ be a Normally distributed random variable with mean $\mu$ and variance $\sigma^2$.

- $Z = (X-\mu)/\sigma$ follows a “Standard” Normal distribution $N(0,1)$
- or
- $X = \mu + \sigma Z$ with $Z$ following a standard $N(0,1)$ distribution.

Some “classics” in statistics (cont.)

Let $X$ be a Normally distributed random variable with mean $\mu$ and variance $\sigma^2$.

Then:

$$\Pr[X \geq u] = 1 - \Pr[X < u] = 1 - \Phi_{(\mu, \sigma)}(u)$$

$$\Leftrightarrow \Pr\left[\frac{X-\mu}{\sigma} < \frac{u-\mu}{\sigma}\right] = \Pr[Z < \frac{u-\mu}{\sigma}] = \Phi_{(0,1)}\left(\frac{u-\mu}{\sigma}\right)$$
Density and cumulative (normal)

\[ \Phi(K) = \Pr[q \leq K] = \int_{q=0}^{K} f(q) \, dx \]

A simple case: setting the quotas for two fares

Gary Larson: The far side

Early stages of math anxiety
Single flight capacity control

- In practice many RM problems are
  - O-D problems (over a network)
  - Dynamic (demand varies with time and price -> quantity sold varies over time -> prices vary over time -> demand varies with prices ->…)
  - Computationally demanding
- Quite often solved as a collection of single-resources problems

Assumption 1
In the following we will focus on a single flight (no connecting flight or multiple legs)

There is a distinction between “fares classes” and the usually fixed ”transport classes” (first, business, eco,..)

Assumption 2
We will consider a plane with identical seats on board, whatever the price paid (whatever the “class”)

The “fare classes” are only determined by the capacity allowed (or the curtain in the plane)
Single flight capacity control: The two fares case

- One airplane with a fixed configuration $C$ = total capacity
- Two fares $P_L$ (low) and $P_H$ (High)
- The demand distributions for the two classes $L$ and $H$ are assumed to be known $f_L(q)$ and $f_H(q)$.

The two fares case

In this STATIC framework:
What is the optimal number of seats $Q$, sold at price $P_H$?
Distribution of demand: What is the optimal quota Q!

The trade-off

- The number of seats in demand is by nature random
- Let’s consider a demand with mean N (let us assume a normal distribution)
- If one allocate a small quota Q (less than N), there is a risk of rejecting consumers (Spill)
- If one allocate a high quota (more than N), there is a risk of empty seats (spoilage).

➤ Quota allocation is the core of RM
Distribution of demand: Definitions

- Demand
- Quota
- Probability of refusing a sale (spill)
- Probability of having at least one empty seat (spoilage)

Variations on the quota: Situation A

- Probability of having at least one empty seat = 0.5
- Probability of refusing one or more sales = 0.5

Q = NH (here median = mean)
Variations on the quota: Situation B

Pr(x)

Demand

Probability of having at least one empty seat = small
Probability of refusing one or more sales = big !!

Q Small

Variations on the quota: Situation C

Pr(x)

Demand

Probability of having at least one empty seat = big !
Probability of refusing one or more sales = small

Big Q
The two doors example

There are 2 seats available for the course

❖ Behind door A are 10 people ready to pay 10€ to attend
❖ Behind door B are some people ready to pay 100€ to attend, but maybe nobody
❖ We have to choose which door to open

What should we do?

What do we need to decide?

The two doors example: Numerical illustration

❖ We know the probabilities
  ▪ Prob. there is 0 person = 0.5
  ▪ Prob. there is 1 person = 0.25
  ▪ Prob. there are 2 people = 0.15
  ▪ Prob. there are more than 2 people = 0.10
❖ I open door A: I get 2 x 10 = 20 €
❖ I open door B: I get 0 with p. 0.5, 100€ with p. 0.25, 200€ with p. (0.15+0.10)
My expected revenue is 100x0.25 +200x0.25= 75€

SO ?
Silly question of the day:

How to manage the trade-off?

Two class model

- Assume there are 2 classes \((p_H > p_L)\) a fixed capacity \(C\), no cancellation, no overbooking.

Demands for class L and H are estimated.
(and independent)

What is the best initial allocation of seats between the two classes?
Initial allocation when demand is unknown..

- Two fares: High (H) and Low (L)
- One airplane with a fixed configuration (C seats)
- Prices are fixed for H and L classes

\[ p_H = 100 \, \text{€}, \quad q_H \text{ seats} \]
\[ p_L = 50 \, \text{€}, \quad 120-q_H \text{ seats} \]

Ex: C=120

Distribution of demand: example

“low fare” (or “Leisure”) demand (mean \( N_L \)) and “High fare” (or “Business”) demand (mean \( N_H \))
The problem is to compute the value of $Q_H$ such that the global revenue is maximum.

Global Revenue is $R_L + R_H$.

The demand in each class is modeled through its distribution function:
- $f_H$ and $f_L$.
- We assume that the demands are independent.

The global revenue is not deterministic, for each class, one has the expectation of the revenue (linked to the probability of selling a seat = demand distribution).

Global Expected revenue is $= E(R_L) + E(R_H)$ (because we assumed independence of the demands).
Determination of the Quota \((Q_H)\) for two independent classes

- Total expected revenue = \(E(R_H) + E(R_L)\)

\[
E(R_H) = \int_0^{Q_H} P_H \cdot q \cdot f_H(q) \, dq + \int_{Q_H}^{\infty} P_H \cdot Q_H \cdot f_H(q) \, dq
\]

\[
E(R_L) = \int_0^{C-Q_H} P_L \cdot q \cdot f_L(q) \, dq + \int_{C-Q_H}^{\infty} P_L \cdot (C-Q_H) \cdot f_L(q) \, dq
\]

Let's compute the expectation of revenue for the high fares class

\[
E(R_H) = \int_0^Q P_H \cdot q \cdot f_H(q) \, dq = \int_0^{Q_H} P_H \cdot q \cdot f_H(q) \, dq + \int_{Q_H}^{Q} P_H \cdot Q_H \cdot f_H(q) \, dq
\]

Even if demand \(q\) exceeds \(Q_H\), one cannot sell more than \(Q_H\) seats.
What is $Q_H$ for two independent classes?

- $Q_H$ must satisfy the maximization of the expected revenue for all the seats.

$$Q^*_H = \text{Arg max}_{Q_H} \left( E(R_H) + E(R_L) \right)$$

Computation of Quota $Q_H$ for two independent classes

$$\frac{dE(R)}{dQ_H} = \frac{dE(R_H)}{dQ_H} + \frac{dE(R_L)}{dQ_H}$$

$$E[R_H] = \int_0^{Q_H} P_H \cdot q \cdot f_H(q).dq + \int_{Q_H}^{\infty} P_H \cdot Q_H \cdot f_H(q).dq$$

$$\frac{dE(R)}{dQ_H} = P_H \cdot Q_H \cdot f_H(Q_H) - P_H \cdot 0 \cdot f_H(0)$$

$$+ P_H \int_{Q_H}^{\infty} f_H(q).dq + P_H \cdot Q_H \cdot (f_H(\infty) - f_H(Q_H))$$
Computation of Quota $Q_H$ for two independent classes (cont.)

\[
\frac{dE(R_H)}{dQ_H} = P_H \cdot Q_H \cdot f_H(Q_H) - P_H \cdot 0 \cdot f_H(0)
\]

\[+ P_H \int_{Q_H}^{\infty} f_H(q) \, dq + P_H \cdot Q_H \cdot (f_H(\infty) - f_H(Q_H))\]

\[
\frac{dE(R_H)}{dQ_H} = P_H \cdot Q_H \cdot f_H(Q_H) + P_H \int_{Q_H}^{\infty} f_H(q) \, dq - P_H \cdot Q_H \cdot f_H(Q_H)
\]

\[
\frac{dE(R_H)}{dQ_H} = P_H \int_{Q_H}^{\infty} f_H(q) \, dq
\]

Computation for $L$ !

- Doing the same type of computation (to do!)

\[
\frac{dE(R_L)}{dQ_H} = \frac{d}{dQ_H} \left( \int_0^{C-Q_H} P_L \cdot q \cdot f_L(q) \, dq \right)
\]

\[+ \int_{C-Q_H}^{\infty} P_L \cdot (C - Q_H) \cdot f_L(q) \, dq \right) \]

\[= -P_L \int_{C-Q_H}^{\infty} f_L(q) \, dq \]
Finally

\[
\frac{dE(R)}{dQ_H} = P_H \int_{Q_H}^{\infty} f_H(q).dq - P_L \int_{c-Q_H}^{\infty} f_L(q).dq = 0
\]

\[
P_H \int_{Q_H}^{\infty} f_H(q).dq = P_L \int_{c-Q_H}^{\infty} f_L(q).dq
\]

Revenue expected from selling more seat at price \( P_H \)

Definition: EMSR
Expected Marginal Seat Revenue

\[
EMSR_L(S) = \int_{S}^{\infty} P_L \cdot f_L(q).dq = P_L \cdot \left( 1 - \int_{0}^{S} f_L(q).dq \right)
\]

For a high capacity, the expected revenue of an additional seat can be low.
What does (1) mean?

In the case of independent (partitioned fares) classes, at the equilibrium the EMSR must be equal in each class.

\[(1) \iff \text{EMSR}_H(Q_H) = \text{EMSR}_L(C-Q_H)\]

Graphical illustration

- \(\text{EMSR}_L\) = \(\text{EMSR}_H\)
- \(p_Hf_H(x_H)\) and \(P_Lf_L(x_L)\)
- Capacity C

Nathalie LENOIR, October 2011
Graphical illustration

Indifference point:
Here, the expected revenue of a seat is equal in H or L

Remarks

In this simple case, the formula for the optimal quota

\[
p_H \int_{Q_H}^{\infty} f_H(x_H) \, dx_H = p_L \int_{C-Q_H}^{\infty} f_L(x_L) \, dx_L
\]

depends only on

- The distributions \( f_L \) and \( f_H \) of the individual demands in each class
- The prices \( p_L \) and \( p_H \) for each class
Limit of the approach

- The previous approach uses the independence of the demand for the two classes
  - it is true? Only if prices are far apart
- The H fare may be full while there are still empty seats in Leisure..
  - The “low fare seats (leisure)” should be closed to booking before the high class
- The booking behavior is not the same for the business and leisure consumer
  - What if “leisure consumer” take all the seats in advance?
Revenue management: Outline

- **Chap 1: Revenue management basics**
  - Definition, origins and principles
  - Prices and price discrimination
  - Fare classes management

- **Chap 2: Single flight capacity control**
  - A little bit of statistics
  - Setting the quotas for two fares
  - Limits of the approach

- **Chap 3: Revenue management in practice**
  I) Dynamic allocation
  II) Nesting
  III) Network pricing

Revenue management in practice

- **Dynamic allocation**
  - Booking behavior
  - No-show, go-show and over-booking

- **Nesting**
  - Aims and scope
  - Computation
  - Virtual nesting

- **Pricing over a network**
  - Bid prices
  - Virtual classes
Revenue Management In Practice

Extension I: From static to dynamic allocation

Booking behavior

Reservations made

100 %

« Leisure » travellers

Business travellers

Days

Flight reservation opens

Day of departure

Nathalie LENOIR, October 2011
Booking behavior

- The “high fare” passengers reserve their seat later.
  - Schedule change, uncertainty
- The “low fare” book rather in advance
  - Tendency is also linked to restrictions
- The problem is to protect the “high fare” seats until a few days before departure, without losing the “low fare” ones

Managing this trade-off is not simple!

Extension I: From static to Dynamic allocation

- The demands are estimated for each flight, using information on the booking and on past experiences, the computation of $Q_H$ is done using the previous formula.

But...

- The computation has to be revised if the booking behavior shows that the demands are not the ones expected.
- The demands (and $Q_H$) have to be re-estimated using actualized estimations of the demands.
  - In practice, one only revises the allocation if the reservations are not in accordance with the expectations.
General booking behavior over time

Seats booked vs. Day of departure over time for different carriers:
- Long-haul carrier
- Medium-haul carrier
- Short-haul carrier

Booking dynamics

Seats booked vs. Day of departure over time with confidence bounds and mean expected demand.
Booking dynamics

Seats booked

Mean expected demand

Confidence bounds

Real reservations

Time

Day of departure

Warning

Confidence bounds

Real reservations

Time

Day of departure

Nathalie LENOIR, October 2011
New allocation

- When a warning appears, one must re-allocate the seats within each class according to the new (unexpected) demand
  - Revise the demand forecasts
  - Can be done manually or almost automatically

- There may be systems with systematic re-allocation for specific dates (J-90, J-45, J-30…). For each date, one compare the real and expected demand in each class

To summarize:

- One may consider dynamic allocation as a succession of static cases. (not true)

- Demand(s) has (have) to re-estimated for each fare(s)

- Very crude approach of dynamic
No-Show & Go-show

We have assumed that a reserved ticket is a sold ticket, but:

- Not true for tickets with possibility of change in the date of departure, or refundable tickets
- Some people simply do not take the plane they have booked or cancel their reservation at the last minute: «No-shows»
- On the contrary, some people do not reserve in advance and want to fly: «Go-Shows»
“Over-booking”

Used to balance the cancellations and the “no-shows”
- Need to know the distribution of no-shows

Trade-off between two risks
- Risk of empty seats if one accepts few reservations (spoilage)
- Risk of having too many people for the capacity available (denied access)
Figures in a major airline: Go-show & Co

- **No-show = 13% (40-50 000 seats reallocated, each year)**
  - No-shows more frequent for the most demanded flights or on some destinations: South-east of France (cf. *Le Monde* 14.02.07)
  - % of “no show” is decreasing with flight distance
  - Frequent pattern for “business” travelers
- **Go-shows**: Difficult to evaluate

---

**On 10 000 passengers**
- 6-7 denied access
- 6-7 change of class
- Nearly 400 new seats sold

---

**Over-booking “revenue loss”**

![Graph showing the relationship between revenue loss and seats sold with a theoretical optimum at Capacity C.](image-url)

- Revenue loss
- Seats sold
- Capacity C
- Theoretical optimum
- Denied access = cost
- Spoilage = revenue loss

---

*Nathalie LENOIR, October 2011*
Over-booking benefits

Managing denied access

- Usually airline managers are trying to find volunteers for a flight change using financial compensations
- Otherwise, denied access will be applied in priority to “low fare” passengers (difficult in practice)
- The airline must propose a denied access traveler a posterior flight (see Montreal Convention (2004))
Managing denied access

Under EC 261/2004, when an airline has overbooked a flight and therefore cannot accommodate everyone on board, the airline must call on their customers to volunteer not to board that flight in order to free up some seats.

If volunteers come forward they can reach an agreement with the airline as regards compensation. In addition to this agreed compensation the passenger is entitled to look for an alternative flight or a refund of the ticket.

If not enough volunteers come forward the airline can refuse to board passengers but must offer these passengers compensation for their inconvenience. These passengers can claim for €250-€600 depending on the length of their flight and must also be offered an alternative flight or refund of the ticket. These levels of compensation can be reduced under certain circumstances.

Source: European consumer center, Dublin

Denied access: figures


The Bumpiest Rides
Rates in which airlines denied boarding, per 10,000 customers, in the first quarter.

<table>
<thead>
<tr>
<th>Airline</th>
<th>Rate</th>
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<tbody>
<tr>
<td>Delta</td>
<td>3.74</td>
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<tr>
<td>Continental</td>
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<td>US Airways</td>
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<tr>
<td>Average</td>
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<td>0.21</td>
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<tr>
<td>JetBlue</td>
<td>0.04</td>
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</tbody>
</table>

Source: Dept. of Transportation
How to compute the over-booking rate?

- One accepts over-booking (single class case) as long as:
  - The “Expected Marginal Seat Revenue” is greater than the expected marginal cost of a denied access
  - The total cost of denying access for a quota $K > C$ is:
    \[
    P_d \times \sum_{x=C+1}^{C+K} (x - C) \Pr(q = x)
    \]
  - The marginal cost of a denied access is the increase in the cost of denying access (to one or more passengers $q$), from an increase in the quota from $K-1$ to $K$.

How to compute the over-booking rate?

- One has to be able to know the average denied access as a function of the reservation rate and its variance.

- In practice it is quite hard since the «no-shows» are hard to forecast with precision (high variability).

- The cost $P_d$ of a denied access can be high and is “fuzzy” (image, long-term implications, etc..).

- In practice, denied access are moved to higher fare class (upgrade), when possible.
How to compute the over-booking rate?

![Revenue Management Diagram](image)

Accepted reservations

Theoretical optimum

Confidence bounds

Nathalie LENOIR, October 2011

Revenue Management In Practice

Extension II: Nesting
Single flight capacity control: Nested classes

- The pattern of demand for the different classes is different, so the booking mechanism encompass these features
  - A high fare class should not be constrained too much
  - One should always have more seats available in the high fare class
  - One should close the low fare class “before” the high fare class
From independent to nested classes

- One airplane with a fixed configuration $C = \text{total capacity}$
- Two fares $P_L$ (Low) and $P_H$ (High fare)
- The demand distributions for the two classes $L$ and $H$ are assumed to be known $f_L(q)$ and $f_H(q)$.

$\begin{align*}
\text{Disp} & \\
H & 60 \\
L & 40 \\
\text{Tot.} & 100
\end{align*}$
In practice:
Two independent classes (case 1)

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Demand over time

Reservation opens

Flight departure

In practice:
Two independent classes (case 2)

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Demand over time

Reservation opens

Departure

Nathalie LENOIR, October 2011
From Independent to nested classes

- One airplane with a fixed configuration $C = \text{total capacity}$
- Two fares $P_L$ (Low) and $P_H$ (High fare)
- The demand distributions for the two classes $L$ and $H$ are assumed to be known $f_L(q)$ and $f_H(q)$.

In practice:

**Two nested classes (case 2)**

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Demand over time:
- Reservation opens
- Departure

H: 30 seats
L: 10 Seats
Nested case “with protection”

- One find “good” ideas in the literature that may be implemented
  - For example, the “protection” of high fare seats
    “The L class should be closed “before” the H high class”
  - So each ticket sold decreases also the availability of the low fare class.

In practice: two nested classes with protection (case 2)

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Demand over time

Reservation opens

Departure
In practice: Two nested classes with protection (case 2bis)

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<td>L</td>
<td>40</td>
<td>0</td>
</tr>
<tr>
<td>Tot.</td>
<td>100</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Disp</th>
<th>Booked</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>80</td>
<td>10</td>
</tr>
<tr>
<td>L</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>Tot.</td>
<td>80</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Disp</th>
<th>Booked</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>L</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Tot.</td>
<td>40</td>
<td>60</td>
</tr>
</tbody>
</table>

Demand over time

Reservation opens

H: 10 Seats
L: 10 Seats

H: 30 seats
L: 10 seats

Introduction of a booking limit with nested classes

- The basic idea of a booking limit is to avoid to sell a seat with an expected value lower than the value one could have within another class.
  - One protect the higher class (one keep seats for H) until the “expected value of the seat in class H” goes beyond the price of the seat in class L.
    
    \[ i.e. \ K_H \text{ such that } EMSR_H(K_H) \geq P_L \]

- The booking limits (i.e. the capacity \(K\) in each class) depends only of the prices and demand distribution for the two adjacent classes
Graphical computation of the booking limit with two nested classes

\[ EMSR_i(S) = P_i \cdot \left( \int_{S}^{\infty} f_i(q) dq \right) \]

Definitions

Booking limit \( K_H \) vs Protection level \( Y_H \)

Here they coincide!
Computation of the booking limit: $Y_H$

Assume the demand for H class follows a Normal distribution $N(N_H, \sigma_H)$

$Y_H$ is such that $EMSR_H(Y_H) > P_L$

$$EMSR_H(Y_H) = P_H \cdot \int_{Y_H}^{\infty} f_H(q) dq$$

$$EMSR_H(Y_H) \geq P_L$$

$$\Leftrightarrow P_H \cdot \Pr[q_H \geq Y_H] \geq P_L$$

$$\Leftrightarrow \Pr[q_H \geq Y_H] \geq \frac{P_L}{P_H}$$

Computation of the Protection level: $Y_H$

$Y_H$ is such that $EMSR_H(Y_H) > P_L$

$$\Leftrightarrow \Pr_H \left[ \frac{q_H - N_H}{\sigma_H} \geq \frac{Y_H - N_H}{\sigma_H} \right] \geq \frac{P_L}{P_H}$$

$$\Leftrightarrow Y_H \text{ s. t. } \Pr_H \left[ u \geq \frac{Y_H - N_H}{\sigma_H} \right] \geq \frac{P_L}{P_H}$$

Since we assume a Normal distribution for $q_H$, $u$ follows a standard Normal distribution

$$\Leftrightarrow \frac{Y_H - N_H}{\sigma_H} = \Phi^{-1}_H \left( \frac{P_L}{P_H} \right)$$
Computation of the booking limit with four nested classes

- Assume the following (static) distribution of the demand for the four classes

<table>
<thead>
<tr>
<th>Class</th>
<th>Price</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std. dev</td>
</tr>
<tr>
<td>Y</td>
<td>1000€</td>
<td>10</td>
</tr>
<tr>
<td>B</td>
<td>600€</td>
<td>50</td>
</tr>
<tr>
<td>Q</td>
<td>200€</td>
<td>90</td>
</tr>
<tr>
<td>L</td>
<td>100€</td>
<td>100</td>
</tr>
</tbody>
</table>

- And assume that the demand distribution is following a Normal distribution within each class

Computation of the booking limit:

- Assume the capacity is C=200 seats, one wish to know:

<table>
<thead>
<tr>
<th>Class</th>
<th>(initial) Allocation</th>
<th>Protect. Y_i</th>
<th>Booking limit K_i</th>
<th>Price</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Mean</td>
<td>Std. dev</td>
</tr>
<tr>
<td>Y</td>
<td>200</td>
<td>?</td>
<td>?</td>
<td>1000€</td>
<td>10</td>
</tr>
<tr>
<td>B</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>600€</td>
<td>50</td>
</tr>
<tr>
<td>Q</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>200€</td>
<td>90</td>
</tr>
<tr>
<td>L</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>100€</td>
<td>100</td>
</tr>
</tbody>
</table>
Graphical computation of the booking limit ($K_i$) and protection levels ($Y_i$) with four nested classes

Silly question of the day:

What feature of the demand distribution is it important to know here?
Computation of the protection limit: $Y_Y$

$Y_Y$ is such that $\text{EMSR}_Y (Y_Y) > P_B$

$$P_Y \cdot \Pr_Y [q_Y \geq Y_Y] \geq P_B$$

$$\iff 1000 \cdot \Pr_Y [q_Y \geq Y_Y] \geq 600$$

$$\iff \Pr_Y [q_Y \geq Y_Y] \geq 0.6$$

$$\iff \Pr_Y \left[ \frac{q_Y - \bar{Y}_Y - 10}{4} \right] \geq 0.6 \iff Y_Y \text{ s. t. } \Pr_Y [u \geq \frac{Y_Y - 10}{4}] \geq 0.6$$

Since we assume a Normal distribution for $q_Y$, we search $X$ such that for $u$, a $N(0,1)$ random variable

$$\iff X \text{ s. t. } \Pr_Y [u \geq X] \geq 0.6$$

### Normal distribution

<table>
<thead>
<tr>
<th>Class</th>
<th>Price</th>
<th>Mean</th>
<th>Std. dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>1000€</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>$B$</td>
<td>600€</td>
<td>50</td>
<td>10</td>
</tr>
<tr>
<td>$Q$</td>
<td>200€</td>
<td>90</td>
<td>30</td>
</tr>
<tr>
<td>$L$</td>
<td>100€</td>
<td>100</td>
<td>25</td>
</tr>
</tbody>
</table>

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![Normal distribution graph]
Normal distribution : How to read

\[ \phi(0.67) = 0.43 \]

\[ \phi^{-1}(0.6) = 0.25 \]

<table>
<thead>
<tr>
<th>X</th>
<th>0.00</th>
<th>0.01</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
<th>0.05</th>
<th>0.06</th>
<th>0.07</th>
<th>0.08</th>
<th>0.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.5000</td>
<td>0.5040</td>
<td>0.5080</td>
<td>0.5120</td>
<td>0.5160</td>
<td>0.5199</td>
<td>0.5239</td>
<td>0.5279</td>
<td>0.5319</td>
<td>0.5359</td>
</tr>
<tr>
<td>0.1</td>
<td>0.5398</td>
<td>0.5438</td>
<td>0.5478</td>
<td>0.5517</td>
<td>0.5557</td>
<td>0.5596</td>
<td>0.5636</td>
<td>0.5675</td>
<td>0.5714</td>
<td>0.5753</td>
</tr>
<tr>
<td>0.2</td>
<td>0.5793</td>
<td>0.5832</td>
<td>0.5871</td>
<td>0.5910</td>
<td>0.5948</td>
<td>0.5987</td>
<td>0.6026</td>
<td>0.6064</td>
<td>0.6103</td>
<td>0.6141</td>
</tr>
<tr>
<td>0.3</td>
<td>0.6179</td>
<td>0.6217</td>
<td>0.6255</td>
<td>0.6293</td>
<td>0.6331</td>
<td>0.6368</td>
<td>0.6406</td>
<td>0.6443</td>
<td>0.6480</td>
<td>0.6517</td>
</tr>
<tr>
<td>0.4</td>
<td>0.6554</td>
<td>0.6591</td>
<td>0.6628</td>
<td>0.6664</td>
<td>0.6700</td>
<td>0.6736</td>
<td>0.6772</td>
<td>0.6808</td>
<td>0.6844</td>
<td>0.6879</td>
</tr>
<tr>
<td>0.5</td>
<td>0.6915</td>
<td>0.6950</td>
<td>0.6985</td>
<td>0.7019</td>
<td>0.7054</td>
<td>0.7088</td>
<td>0.7123</td>
<td>0.7157</td>
<td>0.7190</td>
<td>0.7224</td>
</tr>
<tr>
<td>0.6</td>
<td>0.7257</td>
<td>0.7291</td>
<td>0.7324</td>
<td>0.7357</td>
<td>0.7389</td>
<td>0.7422</td>
<td>0.7454</td>
<td>0.7486</td>
<td>0.7517</td>
<td>0.7549</td>
</tr>
<tr>
<td>0.7</td>
<td>0.7580</td>
<td>0.7611</td>
<td>0.7642</td>
<td>0.7673</td>
<td>0.7704</td>
<td>0.7734</td>
<td>0.7764</td>
<td>0.7794</td>
<td>0.7823</td>
<td>0.7852</td>
</tr>
<tr>
<td>0.8</td>
<td>0.7891</td>
<td>0.7910</td>
<td>0.7939</td>
<td>0.7967</td>
<td>0.7995</td>
<td>0.8023</td>
<td>0.8051</td>
<td>0.8078</td>
<td>0.8106</td>
<td>0.8133</td>
</tr>
<tr>
<td>0.9</td>
<td>0.8159</td>
<td>0.8186</td>
<td>0.8212</td>
<td>0.8236</td>
<td>0.8260</td>
<td>0.8280</td>
<td>0.8303</td>
<td>0.8320</td>
<td>0.8340</td>
<td>0.8359</td>
</tr>
</tbody>
</table>

Normal distribution (contd)

We search \( X \) such that

\[ \Leftrightarrow X \; s. \; t. \; \Pr[u \geq X] \geq 0.6 \]
\[ \Leftrightarrow X \; s. \; t. \; 1 - \Pr[u < X] \geq 0.6 \]
\[ \Leftrightarrow X \; s. \; t. \; \Pr[u < X] \leq 0.4 \]

Where is it?

- On the negative value of \( X \)!
- And so we have to use the table of the normal distribution using negative \( X \)

If \( X \leq 0 \) Then \( \Pr[u \leq -X] = 1 - \Pr[u \leq X] \)
Computation of the booking limit: $K_Y$

Finally

\[ X \text{ s.t. } -X = \Phi^{-1}(0.6) \iff X = -0.25 \]

\[ \iff Y_Y - 10 = -0.25 \cdot 4 \iff Y_Y = 9 \]

---

Assume capacity is $C=200$ seats, one has:

<table>
<thead>
<tr>
<th>Class</th>
<th>(initial) Allocation</th>
<th>Protect. $Y_i$</th>
<th>Booking limit $K_i$</th>
<th>Price</th>
<th>Demand Mean</th>
<th>Std. dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>200</td>
<td>9</td>
<td>9</td>
<td>1000€</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>191</td>
<td></td>
<td></td>
<td>600€</td>
<td>50</td>
<td>10</td>
</tr>
<tr>
<td>Q</td>
<td></td>
<td></td>
<td></td>
<td>200€</td>
<td>90</td>
<td>30</td>
</tr>
<tr>
<td>L</td>
<td></td>
<td>--</td>
<td>--</td>
<td>100€</td>
<td>100</td>
<td>25</td>
</tr>
</tbody>
</table>
**Computation of the Protection level: \( Y_B \)**

\( Y_B \) is such that \( \text{EMSR}_B > P_Q \)

\[ P_B \cdot \Pr[q_B \geq Y_B] \geq P_Q \]

\( \Leftrightarrow \Pr[q_B \geq Y_B] \geq 0.33 \)

\( \Leftrightarrow \Pr\left[\frac{q_B - 50}{10} \geq \frac{Y_B - 50}{10}\right] \geq 0.33 \)

\( \Leftrightarrow \frac{Y_B - 50}{10} = 0.43 = \Phi^{-1}(0.67) \quad \Leftrightarrow \ Y_B = 4.3 + 50 = 54.3 \]

---

**Computation of the booking limit:**

**Final allocation (static case)**

Assume capacity is \( C=200 \) seats, one has:

<table>
<thead>
<tr>
<th>Class</th>
<th>(initial) Allocation</th>
<th>Protect. ( Y_i )</th>
<th>Booking limit ( K_i )</th>
<th>Price</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y )</td>
<td>200</td>
<td>9</td>
<td>9</td>
<td>1000( \€ )</td>
<td>10</td>
</tr>
<tr>
<td>( B )</td>
<td>191</td>
<td>55</td>
<td>9+55=64</td>
<td>600( \€ )</td>
<td>50</td>
</tr>
<tr>
<td>( Q )</td>
<td>136</td>
<td>--</td>
<td>--</td>
<td>200( \€ )</td>
<td>90</td>
</tr>
<tr>
<td>( L )</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>100( \€ )</td>
<td>100</td>
</tr>
</tbody>
</table>

---

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Computation of the protection limit: $Y_Q$

$Y_Q$ is such that $E\text{MSR}_Q > P_L$

$P_Q \cdot \Pr[q_Q \geq Y_Q] \geq P_L$

$\iff \Pr[q_Q \geq Y_Q] \geq 0.5$

$\iff \Pr\left[\frac{q_Q - 90}{30} \geq \frac{Y_Q - 90}{30}\right] \geq 0.5$

$\frac{Y_Q - 90}{30} = 0 \iff Y_Q = 90$

---

Computation of the booking limit:

Final allocation (static case)

Assume capacity is $C=200$ seats, one has:

<table>
<thead>
<tr>
<th>Class</th>
<th>(initial) Allocation</th>
<th>Protect. $Y_i$</th>
<th>Booking limit $K_i$</th>
<th>Price</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Std. dev</td>
</tr>
<tr>
<td>Y</td>
<td>200</td>
<td>9</td>
<td>9</td>
<td>1000€</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>191</td>
<td>55</td>
<td>9+55=64</td>
<td>600€</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Q</td>
<td>136</td>
<td>90</td>
<td>64+90=154</td>
<td>200€</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>46</td>
<td>--</td>
<td>--</td>
<td>100€</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

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Booking limit with four nested classes

Time interpretation
Definition : EMSR-a

- This method consisting in adding the booking limits in each class to the other booking limits already defined is called EMSR-a “can be” suboptimal (see example)

- EMSR-b should work better (still in debate though) the aggregated future demand (i.e. for upper classes) is considered for determining the protection level

Syntax : EMSR-b

Consider class j+1 (there are many classes, 1 is the highest)
Define the aggregated demand for classes upper classes

\[ S_j = \sum_{k=1}^{j} q_k \]

And define the weighted average revenue from upper classes:

\[ p_j^* = \frac{\sum_{k=1}^{j} p_k E[q_k]}{\sum_{k=1}^{j} E[q_k]} \]

Then one use the stopping rule for defining \( K_i \)

\[ P_j^*. \ P(S_j > K_j) = P_{j+1} \]
### Numerical example 1

<table>
<thead>
<tr>
<th>Class</th>
<th>Price</th>
<th>Demand</th>
<th>Protection levels Y&lt;sub&gt;i&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>Std. dev</td>
</tr>
<tr>
<td>1</td>
<td>1050€</td>
<td>17.3</td>
<td>5.8</td>
</tr>
<tr>
<td>2</td>
<td>950€</td>
<td>45.1</td>
<td>15.0</td>
</tr>
<tr>
<td>3</td>
<td>699€</td>
<td>39.6</td>
<td>13.2</td>
</tr>
<tr>
<td>4</td>
<td>520€</td>
<td>34.0</td>
<td>11.3</td>
</tr>
</tbody>
</table>

Source Talluri – van Ryrin

### Numerical example 2

<table>
<thead>
<tr>
<th>Class</th>
<th>Price</th>
<th>Demand</th>
<th>Protection levels Y&lt;sub&gt;i&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>Std. dev</td>
</tr>
<tr>
<td>1</td>
<td>1050€</td>
<td>17.3</td>
<td>5.8</td>
</tr>
<tr>
<td>2</td>
<td>567€</td>
<td>45.1</td>
<td>15.0</td>
</tr>
<tr>
<td>3</td>
<td>534€</td>
<td>39.6</td>
<td>13.2</td>
</tr>
<tr>
<td>4</td>
<td>520€</td>
<td>34.0</td>
<td>11.3</td>
</tr>
</tbody>
</table>

Source Talluri – van Ryrin

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Remarks

- When assuming independent classes, there is no need to have assumptions about the order of the booking only the total demand matters.
- When using nested classes, since a sale in class $i$ impacts the availability of seats in the other classes, the order of the booking affects the results (early bird assumption).
- The analysis done in the previous slides lies on a static framework (static evaluation of the demand for each class). The “booking limits” are “a priori” booking limits and not really operational ones.

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Revenue Management In Practice

Extension III: Pricing over a network
Bid prices: Another approach

- The control variable is not the quota within each class but a “bid price”
  - Only one variable to store at a given time
- The bid price can be defined as the cost of consuming (selling) the next unit of capacity (seat)
  - It is the opportunity cost of capacity: if I do not sell it now, how much revenue could I expect from it?
  - So, it is the expected revenue for the first seat still available
  - Function of the current remaining capacity

Bid Prices: how does it work?

- One accepts the transaction if the fare is above the bid price. If not, one does not accept the transaction for that fare (fare class is closed)
  - Revenue-based” controls instead of “class-based”
  - Fare (real revenue from selling seat) should be at or above expected marginal revenue
- Dynamic process over time, bid price is computed after each seat sold (reserved)
  - And increases in general as available capacity decreases (unless demand does not reach capacity)
Bid prices

- When reservation opens with capacity $C$, and for a single fare demand

$$ Bid_L(C) = EMSR(C) = P_L \cdot \int_C^{\infty} f_L(q) dq $$

- After $N$ seats sold, the expected value of the last seat sold is the bid price for $C-N$

$$ Bid_L(C - N) = P_L \cdot \int_{C-N}^{\infty} f_L(q) dq $$

(Remember that $0 < C-N < C$)

Bid Prices: graphical illustration

- Graph showing the bid prices $Bid(C)$ and $Bid(C-N)$ with $EMSR_L$ as the expected marginal seat revenue.
Bid prices (Several fares)

- With several fares classes (B,Q,L), the principle is to **sell tickets in class L** until the fare of the Q class is > Bid price_L
- When Bid price becomes > fare in the upper class, one “closes” L class and sell Q class tickets, etc….
- Bid Prices are increasing with seat sold
- Low fare seats are available first then the class is closed
- The problem is a little bit more complicated, since one must then compute the demand for (Q+B), to determine the bid price for the residual demand

Remarks

- “Dual” approach to the capacity constrain approach
- If many fares, one has a smooth increase of prices as the seats are sold.
- Price increases as the remaining capacity decreases
- One may add prices over O-D legs (while it is very difficult to sum quotas for each leg…
  - Adapted to RM over a network
Revenue Management over a Network

Source: Aaron Koblin (Traffic Us)
http://www.aaronkoblin.com/work/flightpatterns/

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Network pricing

✈️ A bit more complicated…

Capacity $Q_1 = (Q_{B1}, Q_{L1})$
Prices $(P_{B1}, P_{L1})$

$Q_2 = (Q_{B2}, Q_{L2}), \ P_2 = (P_{B2}, P_{L2})$

TLS → CDG → LCY
LIS

$Q_3 = (Q_{B3}, Q_{L3}), \ P_3 = …$

$Q_4 = (Q_{B4}, Q_{L4}), \ P_4 = …$

LAI

Nathalie LENOIR, October 2011
Network pricing

- **Create virtual Classes (A, B, C, D) corresponding to Origin-Destination journeys**

  Capacity $Q = (Q_{B1}, Q_{L1}, Q_{B2}, Q_{L2}, Q_{B3}, Q_{L3}, Q_{B4}, Q_{L4}, ...)$ constrained

  - **A** to **CDG**
  - **B** to **LCY**
  - **C** to **LIS**
  - **D** to **LAI**

  **Plus..**

  - **E** to **LCY**
  - **F** to **LIS**
  - **G** to **LAI**
  - **H** to **LIS**
  - **I** to **LAI**
  - **J** to **LIS**

Nathalie LENOIR, October 2011
Some facts

- A major airline with 220 destinations
- 35,000 real O&D offered
  - 2 ways (departure return)
    - 20 fare classes
- Millions of O&D fares classes

On top of that, there may be different ways of doing the same trip if multiple hubs (ex: KLM- Air France)

Network problems

- Many constrains (interdependence among the resources)
- Many different demands
- Overlapping demands
- Complexity of the network
- Problems of multiple Hubs..
- Revenue Integrity

→ Need to jointly manage (coordinate) the capacity controls on all resources
Revenue integrity

Examples of problems:
- Canceled reservations
- Cross border (only one leg is used)
- Go-show with low price
- False 3rd class discrimination (exchange of tickets..)

Losses estimated at 8-10% of the revenue

O&D Revenue Management

Main ideas:
- The O-D fares are ranked according to their expected revenue
  - Pb: what are the demands?
- Priority for reservation are given following this control scheme
  - If only one leg is capacity constrained, the priority is given to long-range travelers paying more than local travelers
  - If all legs are constrained, priority is given to local travelers with comparison of the sum of fares paid
Using demand on each of the O-D defined by virtual classes (A, B, C, D), compute the EMSV to determine the corresponding quota within each virtual class.

Use nested allocation with seat protection and constrains (the leg TLS-CDG is common) to solve the problem.

\[
\text{Capacity } Q = (Q_{B1}, Q_{L1}, Q_{B2}, Q_{L2}, Q_{B3}, Q_{L3}, Q_{B4}, Q_{L4}, \ldots)
\]

RM over a Network: Bid prices

Main ideas:
- Compute the remaining capacity on each leg and compute bid prices.
- Compute the bid price for an O-D as the sum of the bid prices for each leg.
- Then refuse a sale if the price of the current leg is smaller than the current value of the bid price.
- And so refuse a sale if total price (the sum of prices for each leg) is smaller than O-D bid price (the sum of the bid prices for each segment).
Example

TLS  CDG  LIS

<table>
<thead>
<tr>
<th>Disp</th>
<th>Disp</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>4</td>
</tr>
<tr>
<td>Y</td>
<td>3</td>
</tr>
<tr>
<td>L</td>
<td>2</td>
</tr>
</tbody>
</table>

Bid price for $L = 185€$

Bid price for $L = 300€$

As long the $L$ fare on the TLS-LIS is $> 485€$ one takes passengers and propose a $L$ fare (bid prices change)

Example (bis)

TLS  CDG  LIS

<table>
<thead>
<tr>
<th>Disp</th>
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</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>4</td>
</tr>
<tr>
<td>Y</td>
<td>3</td>
</tr>
<tr>
<td>L</td>
<td>0</td>
</tr>
</tbody>
</table>

Bid price for $L = 185€$

Bid price for $L = 300€$

Still, if the price of the $L$ fare on the TLS-LIS is $> 485€$ one takes passengers and propose a $L$ fare while one refuses $L$ sales on the TLS-CDG…
What to remember

- The overriding logic is simple:
  - Price discrimination is provided by product differentiation
  - The expected marginal seat revenue is at the core of the RM mechanics
  - Demand distributions (or quantiles) are mandatory for implementing RM
  - Comparisons are made “in probability”
  - Many heuristics exist, but few theoretical results
What to remember (another story)

The overriding logic is simple
- Capacity is allocated to a request iff its expected revenue is greater than the value of the capacity required to satisfy it.
- The value of the capacity is measured by its (expected) “displacement cost” or “opportunity cost” i.e. the (expected) loss in future revenue from using the capacity now rather than in the future.

Final remarks

- Some passengers are ready to switch from one class to another (if their first choice is full)
  - The low fares seats have to be booked in advance
  - Always keep seats in higher classes
  - In reality demands are not independent
- One may introduce a probability of accepting a fare $P_B$ if one has been rejected in a $P_L$ fare class
  - Complex statistical computations + estimation of this probability = experimental stage
Final remarks

- The current systems are quite complex, demand is still a random variable
  - There is a cost to such a mechanism (experts, software, management)
  - There is also a cost in making mistakes!! (Denied access, over-booking or empty seats)
- Major airlines propose such a complex mechanism that pricing seems fuzzy to travelers (readability problem)
  - People happy about low prices, but unhappy about a complex pricing system they do not understand

Remarks

- Revenue management has changed the pricing and management of airlines but also the travelers’ behavior
- Some last minute seats are available and people may know that feature
- Booking behavior may be affected by a too complex mechanism
Final remarks: low costs

Low cost airlines propose a simple revenue management scheme

- « Our fares change as seats are sold » Easyjet
- Price increases with time
- Very clear pricing
- Very cheap management system based only on booking dynamics over time
- Still this is revenue management but not based on restrictions
  - very few “no-show” since the tickets are non refundable

Final remarks: low costs (fwd)

- Low costs pricing is not based on discrimination and so the tariff restrictions are the same for every customer
- While in “classic” Revenue Management demand could be separated within customers (“business” vs “leisure”), low cost are not.
- Competition with low cost may be quite tough since “business” travelers may switch to low costs airlines where they are not (so much) discriminated.
Bibliography

- Van Ryzin G. “Airline Revenue Management and e-markets” Colombia University.