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NONLINEAR ANALYSIS OF ELECTRICITY PRICES

Risk Premia in Electricity Forward Prices

Pavel Diko* Steve Lawford† Valerie Limpens‡

*Electrabel S.A., pavel.diko@electrabel.com
†Electrabel S.A., steve.lawford@electrabel.com
‡Electrabel S.A., valerie.limpens@electrabel.com

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Abstract

We investigate the presence of significant electricity forward risk premia, using data from three major continental European energy markets - German, Dutch and French. We introduce the risk premium in the framework of a standard electricity spot/forward unobserved factor model, and derive the implied forward price behaviour. We then assess the term-structure and time-evolution of the risk premia for each of the markets.

*Corresponding author: Pavel Diko. Address: Electrabel, avenue Einstein 2a, 1348 Louvain-la-Neuve, Belgium. Email: pavel.diko@electrabel.com. Diko, Lawford and Limpens are with the Strategy, Research and Development team, Electrabel SA. We acknowledge support from Electrabel SA, Jacqueline Boucher and André Bihain. All remaining errors are our own. This paper was compiled using Miktex, and numerical results were obtained using Eviews and Python.

1 Introduction

Early models of electricity market prices that were inspired by the financial literature generally argued that (discounted) forward prices should equal current spot prices. This no-arbitrage reasoning was based on a buy-and-hold strategy that works well in typical financial markets yet breaks down in electricity markets due to the non-storable nature of the traded commodity. In a sense, electricity forwards are not strictly speaking derivatives, because their value is not a function of another traded asset. However, they are basic tradables in the electricity market, and the presence of a properly defined risk premium in their prices cannot be ruled out.

In this paper, we study the presence of risk premia in three of the most liquid continental European electricity markets: Germany (EEX), France (Powernext: PWN) and the Netherlands (APX). Our analysis follows and extends the results of recent work by several authors. Longstaff and Wang (2004) and Karakatsani and Bunn (2005) both perform nonparametric analysis of very-short-term forward prices, and spot prices, on the American PJM, and UK electricity markets, and find evidence of significant risk premia. Villaplana (2003) calibrates a two-factor mean-reverting model with jumps, using historical electricity spot prices on the PJM market and then compares the theoretical price of the forward with that quoted on the market, to obtain the risk premium in terms of the market price of risk of the underlying risk-factors. The former approach is unable to provide information on the risk premium over longer time horizons, while the latter is sensitive to estimation of seasonality patterns in electricity spot price series (robust estimation of seasonality is typically difficult to perform in electricity markets, see e.g. Culot et al. (2006) for discussion).

Our method combines the advantages of the two approaches to provide estimates of risk premia at various time horizons, while being robust to spot price seasonality. Moreover, we extend the previous work and show an apparent time-evolution of the risk premia, and a clear reduction in magnitude with progressive maturity of the markets. We focus on major European markets - German, French and Dutch - and consider separately peak and off-peak prices, as the properties of risk premia have been shown to differ empirically for peak and off-peak (see Karakatsani and Bunn (2005)). The theoretical equilibrium model of Bessembinder and Lemmon (2002) also predicts different behaviour for peak and off-peak risk premia due to different demand levels in the two periods. The spot price data used in this paper is taken from EEX, PWN and APX, while the forward data is taken from Platts, an independent energy market data publishing company (further details are given below).

The rest of the paper is organized as follows. In Section 2, we perform a nonparametric analysis along the lines of Longstaff and Wang (2004) that shows significant risk premia in short-term forwards in all three markets. We recast the analysis in a dynamic trading framework that allows us to demonstrate the time-evolution of the risk premium. Section 3 introduces a simple forward market model and links the theoretical notion of the market price of risk to the results of Section 2. In Sections 4 and 5, we motivate the use of a three-factor forward market model for real market prices, show that our main empirical results confirm significant risk premia in forward prices, and offer a visualization of the risk premium *term-structure*. Section 6 concludes.

2 Empirical Motivation

To illustrate, we examine the performance of a hypothetical *sliding MWh* trading strategy (discussed in Hinz et al. (2005)) on each of the markets, for both peakload and off-peak hours, and consider short-term forwards (namely, day-ahead over-the-counter prices, as published by Platts), and spot prices (exchange average clearing price). We start with an initial capital $K(0)$, and repeat the following strategy on each day $i \in [1, D]$:

Buy on Forward On day i, invest all current capital $K(i-1)$, in order to buy power on the over-the-counter market (paying the over-the-counter price $F(i)$).

Sell on Spot Also on day i, resell the power on the spot exchange (receiving the exchange clearing price $S(i)$), ending with a new amount of capital $K(i)$.

For instance, if $K(0) = 10€$, and spot and forward prices are $S(1) = 5€/\text{MWh}$ and $F(1) = 4€/\text{MWh}$ respectively, then we will buy $\rho := K(0)/F(1) = 2.5\text{MWh}$ on the forward market, and receive $\rho S(1) = 12.5€$ from sale on the spot market. The accumulated capital at the end of day $d \leq D$ from pursuing this strategy, assuming now that $K(0) = 1$, is given by $K(d) = \Pi_{i=1}^{d} S(i)/F(i)$, so that $\ln K(d) = \sum_{i=1}^{d}\{\ln S(i) - \ln F(i)\}$. A positive (negative) average risk premium over $[1, d]$ corresponds to $\ln K(d)$ greater (less) than zero, i.e. spot price greater (less) than forward price, on average. The accumulated log capital $\ln K(d)$ over time, using actual market data, is plotted in Figures 1–6, for EEX, APX and PWN peakload and off-peak hours.

The spot price $S(i)$ is the mean (conditional on peak or off-peak hour) exchange clearing price for physical delivery of power on day i. Spot prices and volumes on each of the markets are determined in advance by a two-sided blind auction, organized by the individual exchanges. Until the morning of the day prior to delivery, market participants may continuously propose price/quantity bid/sell combinations using an electronic system, for each hour of the delivery day. The bids are entered into a sealed order book, and upon closure of the bidding phase, are aggregated to give market demand and supply curves for the following day. The intersection of each of these curves gives the market clearing price and volume by hour. Peak hours are market-specific, and are given as: [0700-2300) on APX, [0800-2000) on EEX and [0800-2000) on PWN. For further details, see the exchange websites www.apx.nl, www.eex.de and www.powernext.fr.

The forward price $F(i)$ is determined on an over-the-counter (bilateral) market, on day $i-1$, also for delivery on day i. The sample periods are from 01/2001 to 08/2005, noting that there are no forward quotations on weekends. Returns are calculated relative to the next trading day. While exchanges give 24 hourly spot prices, set on day $i-1$ for delivery on i, Platts (see www.platts.com for details) gives base, peak and off-peak flat delivery forward prices, where peak/off-peak hours are the same on the exchanges and the over-the-counter market. The Platts prices are a volume-weighted mean of all over-the-counter deals for delivery on day i executed during day $i-1$. There are no geographical mismatches between the exchange and over-the-counter markets (as may arise on American markets), nor are there any timing mismatches (since virtually all over-the-counter deals are executed before the spot exchange closes on day $i-1$).

Summary descriptive statistics for $R(i) := \ln S(i) - \ln F(i)$ are given in Table 1, where D is the number of observations (not necessarily consecutive days), $\mathrm{E}[R]$ and s.d.$[R]$ are the estimated mean and standard deviation, and τ is the $t-$statistic that E$[R]= 0$. Clearly, there is a very significant negative average short-term risk premium for all markets, during peakload hours, over the sample period. We observe a positive average short-term risk premium on EEX off-peak hours, that is significant at the 5% level. There is no significant average risk premium during either APX or PWN off-peak hours.

	EEX		APX		PWN	
	peak	off-peak	peak	off-peak	peak	off-peak
D	721	846	698	833	717	842
E$[R]$	−0.04211	0.01325	−0.04656	0.00732	−0.02232	0.00545
s.d.$[R]$	0.19849	0.16361	0.21905	0.24384	0.14428	0.12141
τ	−5.70	2.36	−5.62	0.87	−4.14	1.30

Table 1: Summary descriptive statistics for $R(i) := \ln S(i) - \ln F(i), i \in [1, D]$, where $R(\cdot)$ is the one-day risk premium, $S(\cdot)$ and $F(\cdot)$ the spot and forward prices respectively, i the day (observation number), and D the total number of observations in the sample. The sample mean (and corresponding t-statistic, τ) and sample standard deviation, are reported for peak and off-peak hours, on the EEX, APX and PWN power markets.

Note that the slope of $\ln K(d)$ represents the *instantaneous* short-term (one-day) risk premium, which can be approximated on day d by $\Delta \ln K(d)/\Delta d$, which equals $\ln K(d) - \ln K(d-1) = \ln S(d) - \ln F(d) = R(d)$, given $\Delta^k := 1 - L^k$, with L the lag operator, and $k = 1$. Further, $(1/2)[\ln K(d) - \ln K(d-2)] = (1/2)[R(d) + R(d-1)]$, if $k = 2$, and an $m-$step backwards moving average of $R(d)$ when $k = m$. A more accurate approximation, which is convenient from a graphical viewpoint, is given by constructing a univariate Nadaraya-Watson kernel regression estimator of $R(i)$ on i (days). The kernel estimator of $R(x)$ at every point x is given by

$$\widehat{R}(x) = \arg\min_{\psi} \sum_{i=1}^{D} (R(i) - \psi)^2 K((x-i)/h),$$

where D is the number of observations in the sample, and ψ is a locally-fit constant. The bandwidth h controls the degree of smoothing. It is fixed across the sample, and is selected by the "rule-of-thumb" $h = 0.15(\max\{i\} - \min\{i\}) = 0.15(D-1)$. The kernel weighting function $K(u)$ is chosen to be the Gaussian density function $(2\pi)^{-1/2}\exp(-u^2/2)$. For an introduction to kernel techniques, see Abadir and Lawford (2004) and references therein. The results are plotted in Figures 7–12, and give a clear indication of time-varying trends in the instantaneous short-term risk premia.

3 Single-Factor Market Model

Here, we introduce the concept of the risk premium in a standard theoretical model of the spot and forward electricity market, that has been analyzed by, inter alia, Clewlow and Strickland (1999). Define a filtered probability space (Ω, F, F_t, P), where F_t is a (null-set augmented) filtration generated by a one-dimensional Wiener process W_t, and P the physical measure. We denote by $S_t = \exp(X_t)$ the *spot price* of electricity for delivery at time t. The time evolution of the log-spot price follows an Ornstein-Uhlenbeck process with constant speed of reversion α, instantaneous volatility σ and time-varying mean level β_t, satisfying the stochastic differential equation (SDE)

$$dX_t = \alpha(\beta_t - X_t) + \sigma dW_t, \quad \alpha, \sigma > 0.$$

The price of a forward contract at time t for delivery at time $T \geq t$ will be denoted $F(t,T)$. We assume that at each time t, a forward contract with every maturity $T \geq t$ can be traded. As shown by Clewlow and Strickland (1999), in the absence of arbitrage the dynamics of the forward price $F(t,T)$ under the (unique) risk-neutral measure Q equivalent to P, satisfy the SDE

$$\frac{dF(t,T)}{F(t,T)} = \sigma e^{-\alpha(T-t)} d\widetilde{W}_t, \tag{3.1}$$

where $\widetilde{W}_t$ denotes a Wiener process under Q. The risk-neutral dynamics of the wealth K_t generated by the continuous time equivalent of the *sliding MWh* strategy in this model are shown in the Appendix to follow

$$\frac{dK_t}{K_t} = \sigma d\widetilde{W}_t.$$

As expected, the wealth process of this trading strategy is a Q-martingale. However, our preliminary empirical observations show that under the historical probability measure P, the wealth process K_t has a (negative) drift. So, under P we observe that

$$\frac{dK_t}{K_t} = \nu dt + \sigma dW_t.$$

From the Cameron-Martin-Girsanov theorem, we have that $d\widetilde{W}_t = dW_t + (\nu/\sigma)dt$, and by substituting into (3.1) we obtain the implied forward dynamics under the physical measure:

$$\frac{dF(t,T)}{F(t,T)} = \nu e^{-\alpha(T-t)} dt + \sigma e^{-\alpha(T-t)} dW_t.$$

We conclude that in the simple one-factor mean-reverting market model the empirically observed drift in the *sliding MWh* trading strategy should imply a drift (decreasing with time to maturity) in the forward contracts. In Section 5, we directly extend the above argument to a more realistic market model.

3.1 Estimation

In practice, electricity markets quote forwards for delivery over a period of time (for instance, a week, month, quarter, or year). Let $\overline{F}(t, T_1, T_2)$ be the price of a forward contract quoted at time t for delivery over the period $[T_1, T_2]$. To exclude arbitrage we must have

$$\overline{F}(t, T_1, T_2) = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} F(t, s) ds.$$

For the sake of computationally tractable estimation, we approximate the forward price $F(t, T_1, T_2)$ by assuming that the prices of the forward contracts for a single time delivery within the period $[T_1, T_2]$ are approximately equal, i.e. for $s, u \in [T_1, T_2]$: $F(t, s) \approx F(t, u)$. Application of Itô's lemma and rearranging then gives

$$\begin{aligned} \frac{dF(t, T_1, T_2)}{F(t, T_1, T_2)} &= c(t, T_1, T_2, \alpha)(\nu dt + \sigma dW_t), \\ c(t, T_1, T_2, \alpha) &= \frac{e^{-\alpha(T_2 - t)}(1 - e^{(T_2 - T_1 + 1)\alpha})}{(T_2 - T_1)(1 - e^{\alpha})}, \end{aligned}$$

where $c(\cdot)$ is derived by infinite series expansions of $(T_2 - T_1)^{-1} \sum_{T=T_1}^{T_2} \exp(-\alpha(T - t))$, that arises in $(dF(t, T_1, T_2)/F(t, T_1, T_2))/(dK_t/K_t)$. Directly, we obtain the following system:

$$\frac{dK_t}{K_t} = \nu dt + \sigma dW_t, \tag{3.2}$$

$$\frac{dF(t, T_1, T_2)/F(t, T_1, T_2)}{dK_t/K_t} = c(t, T_1, T_2, \alpha). \tag{3.3}$$

The parameters ν and σ are estimated by least squares on (3.2), and α is estimated by minimizing the mean squared error of the difference between the left and right hand sides of (3.3). Note that (3.2) may be written, after use of Itô's lemma, as $d \ln K_t = (\nu - \sigma^2/2)dt + \sigma dW_t$. Discretizing (with one unit of time between consecutive observations) gives $\Delta \ln K_t = (\nu - \sigma^2/2) + \sigma \varepsilon_t$, where ε_t is N(0,1). Estimation of ν and σ follows by maximum likelihood (least squares). The estimation results obtained for the German, French and Dutch markets are in Table 2. They are very similar to the initial nonparametric analysis in Section 2 (indeed, compare Tables 1 and 2), which is unsurprising since a one-factor model only gives enough flexibility to model the strong short-term risk premium.

The numerical estimation of α in $c(t, T_1, T_2, \alpha)$ is constrained such that $\alpha \in [0, 2]$, due to a flat likelihood surface for $\alpha > 2$. For the EEX and APX peak products, $\widehat{\alpha} = 2$, with corresponding half-life 0.35 days. The one-factor model illustrates the focus on the short-term risk premium, and the main insight is that the half-lives on EEX, APX and PWN peak products are all less than half a day. This analysis is extended through the three-factor model below, where we did not face the same numerical problems, and the estimation is unconstrained.

4 Principal Components Analysis

Following Clewlow and Strickland (2000), we perform a principal components analysis on the empirical (symmetric) covariance matrix Σ of $d \ln F/F$ for each market, and for

	EEX		APX		PWN	
	peak	off-peak	peak	off-peak	peak	off-peak
$\widehat{\nu}$	**−0.04211**	**0.01325**	**−0.04656**	0.00732	**−0.02232**	0.00545
(p-value)	1.22×10^{-8}	0.01851	1.96×10^{-8}	0.38620	3.43×10^{-5}	0.19290
$\widehat{\sigma}$	0.19849	0.16361	0.21905	0.24384	0.14428	0.12141
$\widehat{\alpha}$	2	0.202	2	0.278	1.59	0.164
half-life	0.35	3.43	0.35	2.49	0.44	4.23

Table 2: Estimated parameters of the one-factor model. See Section 3.1 and equations (3.2) and (3.3) for further details. The parameters ν and σ are estimated by least squares (the p-value follows directly), and α by nonlinear least squares. The continuous-time half-life is calculated as $(\ln 2)/\alpha$, and is reported in days. Results are reported for peak and off-peak hours, on the EEX, APX and PWN power markets.

peak and off-peak hours. Since the eigenvectors associated with distinct eigenvalues of a normal square matrix ($\Sigma\Sigma' = \Sigma'\Sigma$) are orthogonal, we may reduce the dimension of $(d \ln F/F)$ by sorting eigenvalues in decreasing order of magnitude, and then selecting only those eigenvectors which contribute "substantially" to explaining the observed variation in $d \ln F/F$. This gives a guide to determining the appropriate number of risk-factors that are needed. Results on eigenvalues and eigenvectors are given in Tables 4–15, where elements of eigenvectors that exceed 0.15 in absolute value are highlighted. Notationally, we refer to forward products as (e.g.) EEX_D1P, i.e. the day-ahead EEX peak forward. Other products are W# (week), M# (month), Q# (quarter) and Y# (year), where # denotes a period relative to today, e.g. W1 (forward covering next week), and Q2 (forward covering quarter *after* next). Peak and off-peak products are denoted by P and OP. For EEX, APX and PWN peak hours, we see that three largest principal components explain 95.9%, 97.3% and 96.3% of the variation respectively. For EEX, APX and PWN off-peak hours, the values are reduced to 89.2%, 91.6%, and 87.5%.

The eigenvectors associated with the three largest principal components have a useful interpretation. For instance, on EEX peak hours (Table 5), we see that the first principal component corresponds to short-term effects, through the day-ahead forward. The second and third principal components correspond to medium-term effects (week-ahead and one-month-ahead and two-month-ahead forwards) and longer-term effects (week-ahead up to three-year-ahead forwards), respectively. Similar results are seen for both APX and PWN peak hours, and for off-peak hours, although the impact of the second and third principal components at longer time horizons is then reduced. We conclude that a three-factor model is a sensible improvement over the simple one-factor model outlined above (at least when considering risk premia), where the three factors correspond to short-term, medium-term, and longer-term driving forces. We develop this below.

5 Multi-Factor Market Model

We extend the simple market model of Section 3 by introducing three risk-factors that drive the electricity spot price. Again, we start with a filtered probability space (Ω, F, F_t, P) but now the filtration F_t is generated by 3-dimensional Wiener process

$W_t = (W_t^1, W_t^2, W_t^3)$. Multi-factor models have been proposed by several authors as an appropriate set-up for energy commodities (see e.g. Culot et al. (2006), Koekebakker and Ollmar (2005) and Schwartz and Smith (2000) for discussion). In this framework, the spot price $S_t = \exp(\beta_t + X_t^1 + X_t^2 + X_t^3)$, and under the physical measure P the three risk factors satisfy

$$dX_t^i = -\alpha_i X_t^i dt + \sigma_i dW_t^i, \quad i = 1, 2, 3.$$

The forward contracts under the risk-neutral measure follow

$$\frac{dF(t,T)}{F(t,T)} = \sigma_1 e^{-\alpha_1(T-t)} d\widetilde{W}_t^1 + \sigma_2 e^{-\alpha_2(T-t)} d\widetilde{W}_t^2 + \sigma_3 e^{-\alpha_3(T-t)} d\widetilde{W}_t^3,$$

and by the same reasoning as above, the risk-neutral *sliding MWh* strategy wealth process satisfies

$$\frac{dK_t}{K_t} = \sigma_1 d\widetilde{W}_t^1 + \sigma_2 d\widetilde{W}_t^2 + \sigma_3 d\widetilde{W}_t^3,$$

and under the physical measure:

$$\frac{dK_t}{K_t} = (\nu_1 + \nu_2 + \nu_3)dt + \sigma_1 dW_t^1 + \sigma_2 dW_t^2 + \sigma_3 dW_t^3.$$

The implied historical forward dynamics are then

$$\begin{aligned} \frac{dF(t,T)}{F(t,T)} &= (\nu_1 e^{-\alpha_1(T-t)} + \nu_2 e^{-\alpha_2(T-t)} + \nu_3 e^{-\alpha_3(T-t)})dt \\ &+ \sigma_1 e^{-\alpha_1(T-t)} dW_t^1 + \sigma_2 e^{-\alpha_2(T-t)} dW_t^2 + \sigma_3 e^{-\alpha_3(T-t)} dW_t^3. \end{aligned}$$

This formula effectively allows us to use all of the historical forward prices to estimate the potential risk premium in the forward market at various time horizons. It is also noteworthy that the formula does not involve the function β_t appearing in the spot price evolution equation. Hence, our estimation is robust with respect to the seasonality of power prices, which can be rather complex and difficult to model.

5.1 Estimation

As for the simple one-factor model above, we approximate the market quoted forward prices $\overline{F}(t, T_1, T_2)$ to facilitate estimation. By direct application of Itô's lemma, we obtain

$$\frac{dF(t,T_1,T_2)}{F(t,T_1,T_2)} = \sum_{i=1}^{3} c(t, T_1, T_2, \alpha_i)(\nu_i dt + \sigma_i dW_t^i).$$

The stochastic processes Y_t^1, Y_t^2, Y_t^3 follow the SDE's $dY_t^i = \nu_i dt + \sigma_i dW_t^i, i = 1, 2, 3,$ and we can rewrite the system to be estimated as:

$$dY_t^i = \nu_i dt + \sigma_i dW_t^i, \quad i = 1, 2, 3, \tag{5.1}$$

$$\frac{dK_t}{K_t} = \sum_{i=1}^{3} dY_t^i, \tag{5.2}$$

$$\frac{dF(t, T_1, T_2)}{F(t, T_1, T_2)} = \sum_{i=1}^{3} c(t, T_1, T_2, \alpha_i) dY_t^i. \tag{5.3}$$

After discretization, the system (5.1)–(5.3) can be viewed as a state-space model with state and observation vectors $(\Delta Y_t^1, \Delta Y_t^2, \Delta Y_t^3)$, $(\Delta K_t/K_t, \Delta F(t, T_1, T_2)/F(t, T_1, T_2))$. In principle, this can be estimated using the Kalman filter, modified to account for missing data (since we do not observe all the forward prices on all quotation days), and coupled with a parameter space search algorithm as in Cortazar et al. (2003) . Kalman filter estimation is, however, strongly dependent on the choice of the observation error covariance matrix, which in our case has very large dimension and is not easily parameterized. In addition, the parameter space is large (parameters $(\alpha_1, \alpha_2, \alpha_3, \nu_1, \nu_2, \nu_3, \sigma_1, \sigma_2, \sigma_3)$), and it is no simple matter to find a global optimum. To overcome this problem, we design a two-step least squares estimation procedure in the spirit of the one-factor model estimation.

Step 1. Find dY_t^i and α_i that minimize (where $\alpha_1 < \alpha_2 < \alpha_3$)

$$\sum_t \left\{ \left(\frac{dK_t}{K_t} - \sum_{i=1}^{3} dY_t^i \right)^2 + \sum_{[T_1, T_2]} \left(\frac{dF(t, T_1, T_2)}{F(t, T_1, T_2)} - \sum_{i=1}^{3} c(t, T_1, T_2, \alpha_i) dY_t^i \right)^2 \right\}.$$

Step 2. Given dY_t^i from Step 1, obtain least squares estimates of ν_i and σ_i.

Intuitively, the method first finds $(\alpha_1, \alpha_2, \alpha_3)$ and corresponding $(\Delta Y_t^1, \Delta Y_t^2, \Delta Y_t^3)$ to minimize the model pricing error, and then estimates $(\nu_1, \nu_2, \nu_3, \sigma_1, \sigma_2, \sigma_3)$ as the mean and standard deviation of $(\Delta Y_t^1, \Delta Y_t^2, \Delta Y_t^3)$. In the Appendix, we demonstrate that this estimator is in fact equivalent to the Kalman filter. Parameter estimates for each of the markets are listed in Table 3.

Of particular note, we see that the half-lives associated with the short-term factor (peak products) are all roughly half a day (and $\alpha < 2$), and so the one-factor model *overstates* the short-term rate of reversion (see the discussion in Section 3.1).

5.2 Term-structure and temporal dynamics of risk premia

Using the estimation results from Table 3, we can visualize the *term-structure* of the risk premia; that is, the dependence of the instantaneous drift in the forward prices on the time to maturity. Figures 13–18 plot these for the three markets under study. The results obtained are in very good agreement with the theoretical model of Bessembinder and Lemmon (2002), who predict negative risk premia caused by the presence of skewness in the spot price distribution, and positive risk premia due to the level of volatility in the spot price. As the time to maturity decreases, so does the long-term and medium-term

	EEX		APX		PWN	
Long	peak	off-peak	peak	off-peak	peak	off-peak
$\widehat{\nu}$	0.00115	0.00093	−0.00068	4.96×10^{-5}	0.00071	0.00133
(p-value)	0.05702	0.08980	0.60414	0.98622	0.34245	0.32479
$\widehat{\sigma}$	0.01702	0.01285	0.03379	0.07489	0.01967	0.03519
$\widehat{\alpha}$	0.005	0.00001	0.01	0.005	0.002	0.005
half-life	138.63	69314.72	69.31	138.63	346.57	138.63
Medium	peak	off-peak	peak	off-peak	peak	off-peak
$\widehat{\nu}$	0.00079	−0.00309	0.06126	0.02036	0.00229	−0.00115
(p-value)	0.90052	0.73460	0.11878	0.86526	0.60256	0.92568
$\widehat{\sigma}$	0.17721	0.21353	1.01947	3.13130	0.11403	0.32171
$\widehat{\alpha}$	0.1	0.15	0.5	0.25	0.1	0.15
half-life	6.93	4.62	1.39	2.77	6.93	4.62
Short	peak	off-peak	peak	off-peak	peak	off-peak
$\widehat{\nu}$	**−0.02257**	**0.02948**	**−0.08119**	−0.13910	**−0.01429**	0.01392
(p-value)	0.01785	0.00240	0.04543	0.35910	0.03374	0.28698
$\widehat{\sigma}$	0.26831	0.22699	1.05358	3.95803	0.17472	0.3412
$\widehat{\alpha}$	1.5	1.3	1.2	1.7	1.4	1.7
half-life	0.46	0.53	0.58	0.41	0.50	0.41

Table 3: Estimated parameters of the three-factor model. See Section 5.1 and equations (5.1)-(5.3) for further details. The parameters $\alpha_{(i)}$ are estimated through STEP 1 (Section 5.1), jointly with dY_t^i, the unobserved factors, for $i = 1, 2, 3$. The α's are then sorted by increasing magnitude, to give the "long-term" (small α), medium-term and short-term (large α) risk factors. Parameters ν and σ corresponding to each of the risk factors are then unambiguously estimated using least squares, through STEP 2 (Section 5.1), and the p-value follows directly. The continuous-time half-life is calculated as $(\ln 2)/\alpha$, and is reported in days. Results are reported for peak and off-peak hours, on the EEX, APX and PWN power markets.

risk-factor uncertainty and the corresponding positive risk premium, thereby causing an downward drift in forwards. The price skewness premium caused by unexpected price spikes, and represented by the short-term risk-factor in our model, remains present in the peak prices until just before delivery and then quickly disappears, hence the positive drift. It is not present in the off-peak prices as those are not prone to spikes (the only exception may be the Dutch market, where we see evidence of a negative risk premium, even in the short-term off-peak prices – this may either be a spurious result, or can be explained by relatively high skewness of APX off-peak spot prices).

As suggested by Figures 7–12, the *instantaneous* risk premia seem to evolve over time in all three markets. In particular, and as to be expected, they are smaller as markets become more mature and attract more speculators. We also observe that the risk premium exhibits similar behaviour across the markets. It would be interesting to extend our term-structure analysis to a non-constant risk premium setting, and to further compare risk premia across continental markets. We leave this for future work.

6 Conclusions

In this paper, we have investigated the presence and structure of risk premia in forward prices on three major continental European markets – German, French and Dutch. We confirm previous nonparametric results obtained in the literature on the American PJM, and UK markets, and show that the short-term forward prices are not simply the expectation of the spot prices. We further link the presence of a risk premium with the properties of the forward price dynamics at all time-horizons, and use it to extend the risk premium analysis beyond the very short-term. Taking into account all the available historical data, we discover the presence of significant risk premia in the long-term as well as the short-term. We infer the shape of the risk premium term-structure, i.e. the dependence of the risk premium on time to maturity of the forward contract. We find that the term-structure is in agreement with the theoretical model of the electricity market developed by Bessembinder and Lemmon (2002). It reflects the changing balance of two forces that determine the risk premium, namely the sensitivity to skewness of the spot price, and the variability of the spot price. As the time to maturity increases, the influence of skewness becomes relatively less important compared to the variability, and so the risk premium decreases.

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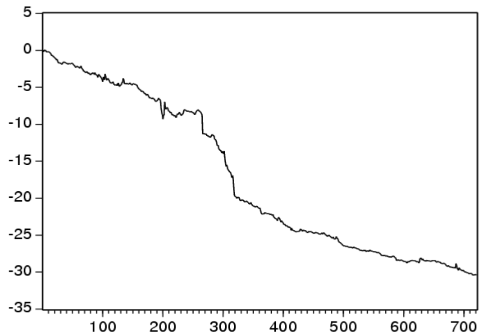

Figure 1: Logarithmic wealth process ($\ln K$) against observation number: EEX peak

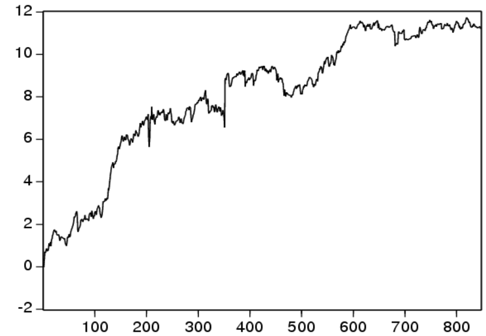

Figure 2: Logarithmic wealth process ($\ln K$) against observation number: EEX off-peak

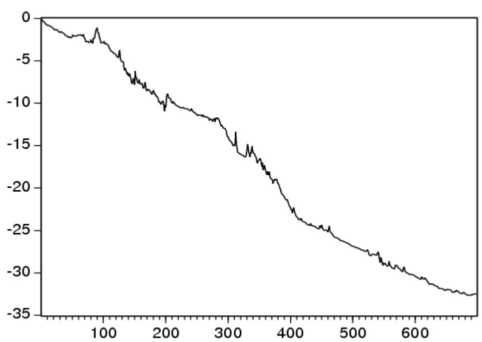

Figure 3: Logarithmic wealth process ($\ln K$) against observation number: APX peak

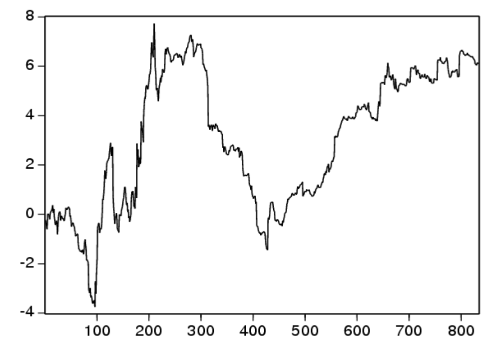

Figure 4: Logarithmic wealth process ($\ln K$) against observation number: APX off-peak

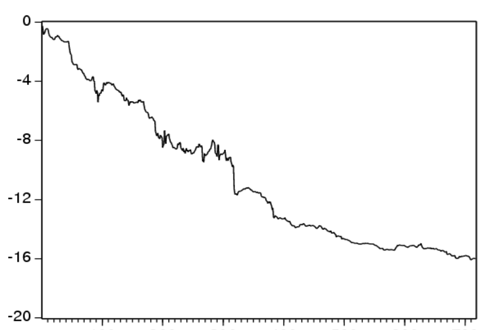

Figure 5: Logarithmic wealth process ($\ln K$) against observation number: PWN peak

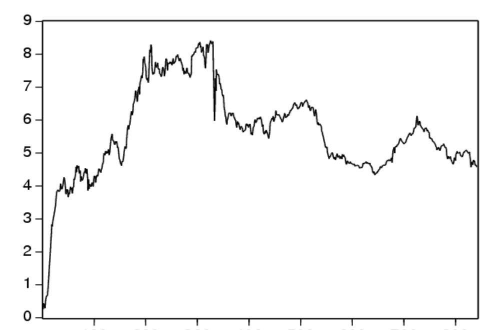

Figure 6: Logarithmic wealth process ($\ln K$) against observation number: PWN off-peak

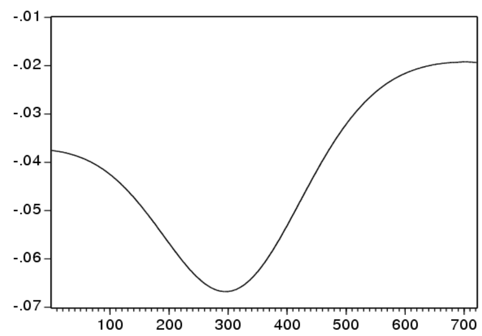

Figure 7: Kernel-fit instantaneous risk premium $\widehat{R}(\cdot)$ ($\times 100$ gives %): EEX peak

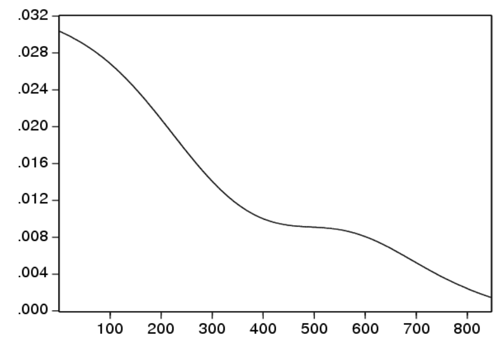

Figure 8: Kernel-fit instantaneous risk premium $\widehat{R}(\cdot)$ ($\times 100$ gives %): EEX off-peak

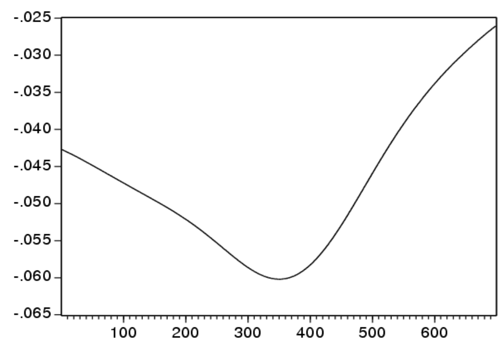

Figure 9: Kernel-fit instantaneous risk premium $\widehat{R}(\cdot)$ ($\times 100$ gives %): APX peak

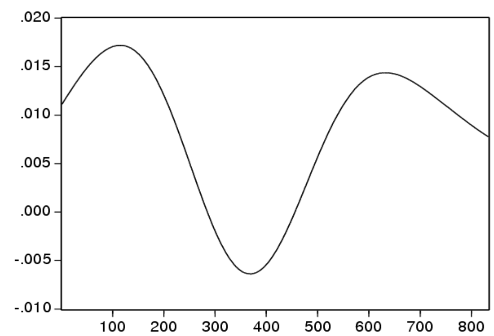

Figure 10: Kernel-fit instantaneous risk premium $\widehat{R}(\cdot)$ ($\times 100$ gives %): APX off-peak

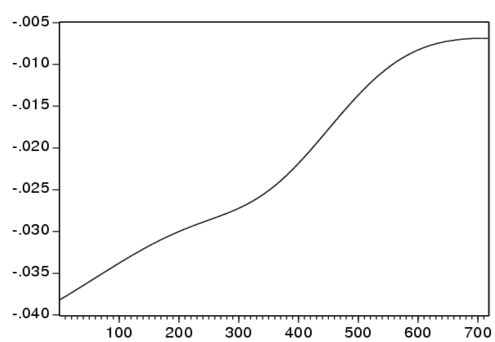

Figure 11: Kernel-fit instantaneous risk premium $\widehat{R}(\cdot)$ ($\times 100$ gives %): PWN peak

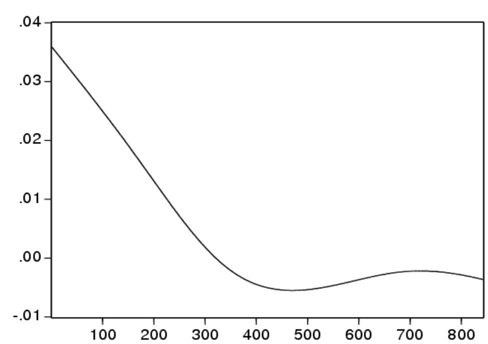

Figure 12: Kernel-fit instantaneous risk premium $\widehat{R}(\cdot)$ ($\times 100$ gives %): PWN off-peak

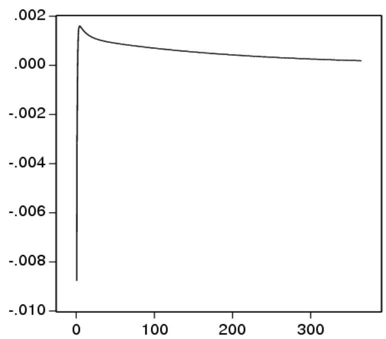

Figure 13: Risk premium term-structure (%/day) (time-to-mat. days): EEX peak

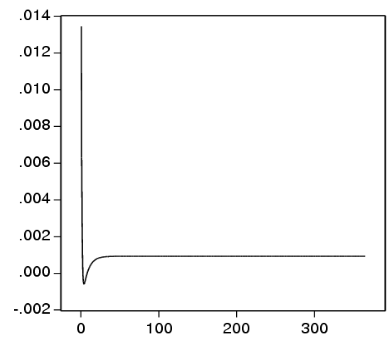

Figure 14: Risk premium term-structure (%/day) (time-to-mat. days): EEX off-peak

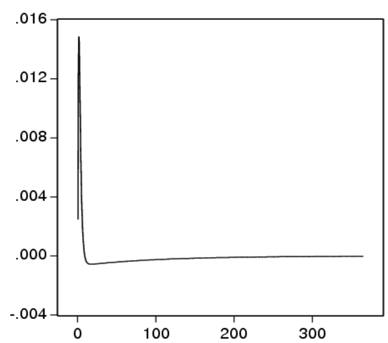

Figure 15: Risk premium term-structure (%/day) (time-to-mat. days): APX peak

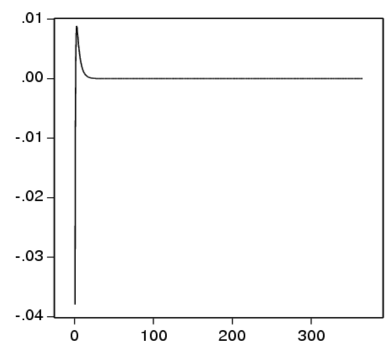

Figure 16: Risk premium term-structure (%/day) (time-to-mat. days): APX off-peak

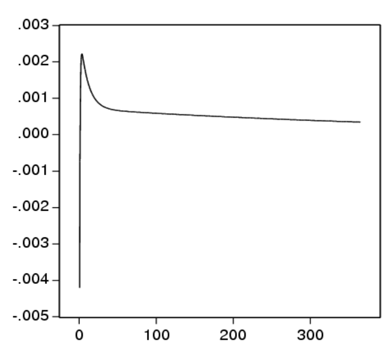

Figure 17: Risk premium term-structure (%/day) (time-to-mat. days): PWN peak

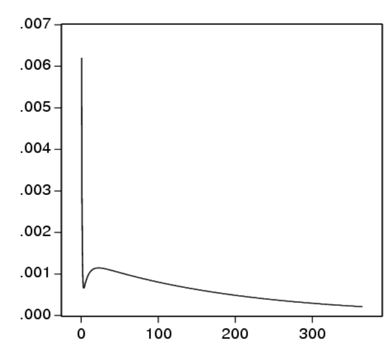

Figure 18: Risk premium term-structure (%/day) (time-to-mat. days): PWN off-peak

EEX peak

Component	Eigenvalue	Variance prop.	Cumulative prop.
Comp 1	0.028918	**0.806624**	0.806624
Comp 2	0.004474	**0.124803**	0.931427
Comp 3	0.000979	**0.02732**	**0.958747**
Comp 4	0.000655	0.018279	0.977026
Comp 5	0.000244	0.006817	0.983843
Comp 6	0.000123	0.00343	0.987273
Comp 7	0.0000992	0.002768	0.990041
Comp 8	0.0000778	0.002171	0.992213
Comp 9	0.0000669	0.001866	0.994079
Comp 10	0.0000578	0.001611	0.99569
Comp 11	0.0000541	0.001508	0.997198
Comp 12	0.0000302	0.000844	0.998042
Comp 13	0.0000278	0.000775	0.998817
Comp 14	0.0000218	0.000608	0.999425
Comp 15	0.0000206	0.000575	1

Table 4: Summary of principal components analysis: Eigenvalues and proportion (cumulative) of variation explained by each component (latter ×100 gives %), EEX peak

Variable	Vector 1	Vector 2	Vector 3
EEX_D1P	**0.991675**	0.126394	−0.014239
EEX_W1P	0.126809	**−0.937227**	**0.187542**
EEX_M1P	0.020925	**−0.25609**	**−0.368589**
EEX_M2P	0.005448	**−0.154066**	**−0.333433**
EEX_M3P	0.002623	−0.078789	**−0.270971**
EEX_M4P	0.000761	−0.006772	**−0.287515**
EEX_M5P	−0.00296	−0.011587	**−0.2266**
EEX_M6P	0.00069	−0.009759	**−0.189085**
EEX_Q1P	0.0022	−0.062955	**−0.354689**
EEX_Q2P	−0.000912	0.00522	**−0.239806**
EEX_Q3P	−0.000996	−0.045609	−0.091033
EEX_Q4P	−0.001788	−0.036149	−0.105014
EEX_Y1P	0.000929	0.015394	**−0.232396**
EEX_Y2P	−0.002161	0.028415	**−0.191589**
EEX_Y3P	−0.000804	0.037517	**−0.425509**

Table 5: Summary of principal components analysis: Eigenvectors corresponding to the three largest eigenvalues (products explained in Section 4), EEX peak

EEX off-peak

Component	Eigenvalue	Variance prop.	Cumulative prop.
Comp 1	0.019391	**0.687414**	0.687414
Comp 2	0.003903	**0.138377**	0.825792
Comp 3	0.001868	**0.06622**	**0.892012**
Comp 4	0.000697	0.024717	0.916728
Comp 5	0.000421	0.014909	0.931637
Comp 6	0.000386	0.013695	0.945332
Comp 7	0.000304	0.010762	0.956094
Comp 8	0.000271	0.009612	0.965706
Comp 9	0.000237	0.008397	0.974103
Comp 10	0.000176	0.006232	0.980335
Comp 11	0.000143	0.005058	0.985393
Comp 12	0.000121	0.004306	0.989699
Comp 13	0.000108	0.003841	0.99354
Comp 14	0.000105	0.003724	0.997264
Comp 15	0.0000772	0.002736	1

Table 6: Summary of principal components analysis: Eigenvalues and proportion (cumulative) of variation explained by each component (latter ×100 gives %), EEX off-peak

Variable	Vector 1	Vector 2	Vector 3
EEX_D1OP	**0.995054**	−0.089808	−0.03918
EEX_W1OP	0.094304	**0.965934**	**0.181016**
EEX_M1OP	0.015469	0.06809	**0.262071**
EEX_M2OP	0.017821	−0.020692	**0.366053**
EEX_M3OP	0.009045	−0.065764	**0.316446**
EEX_M4OP	0.004256	−0.103131	**0.303805**
EEX_M5OP	−0.002925	−0.038423	**0.204193**
EEX_M6OP	−0.001416	−0.053985	**0.168283**
EEX_Q1OP	0.00344	−0.093916	**0.417884**
EEX_Q2OP	−0.00329	−0.045614	**0.238909**
EEX_Q3OP	0.003313	0.016259	0.115969
EEX_Q4OP	0.011723	−0.007383	**0.201498**
EEX_Y1OP	0.003141	−0.110891	**0.306071**
EEX_Y2OP	0.002942	−0.074536	**0.200298**
EEX_Y3OP	0.010787	−0.073603	**0.284638**

Table 7: Summary of principal components analysis: Eigenvectors corresponding to the three largest eigenvalues (products explained in Section 4), EEX off-peak

APX peak

Component	Eigenvalue	Variance prop.	Cumulative prop.
Comp 1	0.045176	**0.782812**	0.782812
Comp 2	0.009612	**0.166558**	0.94937
Comp 3	0.00139	**0.024092**	**0.973462**
Comp 4	0.000596	0.010326	0.983788
Comp 5	0.000359	0.006222	0.99001
Comp 6	0.000198	0.003426	0.993437
Comp 7	0.000143	0.002486	0.995923
Comp 8	0.000129	0.002244	0.998167
Comp 9	0.0000643	0.001114	0.999281
Comp 10	0.0000415	0.000719	1

Table 8: Summary of principal components analysis: Eigenvalues and proportion (cumulative) of variation explained by each component (latter ×100 gives %), APX peak

Variable	Vector 1	Vector 2	Vector 3
APX_D1P	**−0.995422**	−0.094211	−0.000321
APX_W1P	−0.093404	**0.985785**	0.139115
APX_M1P	−0.008911	0.11254	**−0.780492**
APX_M2P	−0.001495	0.065061	**−0.50361**
APX_Q1P	−0.006029	0.037752	**−0.25663**
APX_Q2P	−0.009766	0.01901	**−0.161852**
APX_Q3P	−0.012391	0.012024	−0.11023
APX_Q4P	−0.006009	0.008375	−0.078122
APX_Y1P	0.001691	0.018686	−0.067913
APX_Y2P	−0.002178	0.010199	−0.054176

Table 9: Summary of principal components analysis: Eigenvectors corresponding to the three largest eigenvalues (products explained in Section 4), APX peak

APX off-peak

Component	Eigenvalue	Variance prop.	Cumulative prop.
Comp 1	0.064229	**0.506172**	0.506172
Comp 2	0.044631	**0.351728**	0.8579
Comp 3	0.007344	**0.057874**	**0.915774**
Comp 4	0.004438	0.034973	0.950747
Comp 5	0.001817	0.01432	0.965067
Comp 6	0.001415	0.011149	0.976215
Comp 7	0.001101	0.008673	0.984888
Comp 8	0.000876	0.006904	0.991792
Comp 9	0.000719	0.005662	0.997454
Comp 10	0.000323	0.002546	1

Table 10: Summary of principal components analysis: Eigenvalues and proportion (cumulative) of variation explained by each component (latter ×100 gives %), APX off-peak

Variable	Vector 1	Vector 2	Vector 3
APX_D1OP	**−0.970464**	−0.238448	0.027776
APX_W1OP	**−0.23858**	**0.9696**	−0.011324
APX_M1OP	−0.028456	−0.017072	**−0.975946**
APX_M2OP	0.016435	−0.004221	**−0.211689**
APX_Q1OP	0.002488	−0.005064	−0.031079
APX_Q2OP	−0.01105	0.016967	−0.006574
APX_Q3OP	0.001478	0.007865	−0.018573
APX_Q4OP	0.002619	0.005862	−0.01399
APX_Y1OP	0.007075	−0.047706	−0.003176
APX_Y2OP	−0.003301	−0.004917	−0.016072

Table 11: Summary of principal components analysis: Eigenvectors corresponding to the three largest eigenvalues (products explained in Section 4), APX off-peak

PWN peak

Component	Eigenvalue	Variance prop.	Cumulative prop.
Comp 1	0.019741	**0.731021**	0.731021
Comp 2	0.005354	**0.198251**	0.929272
Comp 3	0.00091	**0.03371**	**0.962982**
Comp 4	0.000404	0.014959	0.977941
Comp 5	0.000259	0.009577	0.987518
Comp 6	0.000189	0.007007	0.994526
Comp 7	0.000148	0.005474	1

Table 12: Summary of principal components analysis: Eigenvalues and proportion (cumulative) of variation explained by each component (latter ×100 gives %), PWN peak

Variable	Vector 1	Vector 2	Vector 3
PWN_D1P	**−0.99371**	−0.106958	0.030252
PWN_W1P	−0.091029	**0.94107**	**0.308796**
PWN_M1P	−0.054421	**0.264436**	**−0.576325**
PWN_M2P	−0.022684	0.142541	**−0.505413**
PWN_Q1P	−0.023795	0.10449	**−0.522919**
PWN_Q2P	−0.010231	0.041393	**−0.179128**
PWN_Y1P	−0.010353	0.008408	−0.102983

Table 13: Summary of principal components analysis: Eigenvectors corresponding to the three largest eigenvalues (products explained in Section 4), PWN peak

PWN off-peak

Component	Eigenvalue	Variance prop.	Cumulative prop.
Comp 1	0.016681	**0.556604**	0.556604
Comp 2	0.007961	**0.265644**	0.822248
Comp 3	0.001567	**0.052291**	**0.874539**
Comp 4	0.001323	0.044151	0.91869
Comp 5	0.001092	0.036435	0.955126
Comp 6	0.000784	0.026149	0.981274
Comp 7	0.000561	0.018726	1

Table 14: Summary of principal components analysis: Eigenvalues and proportion (cumulative) of variation explained by each component (latter ×100 gives %), PWN off-peak

Variable	Vector 1	Vector 2	Vector 3
PWN_D1OP	**−0.995653**	−0.08988	−0.010718
PWN_W1OP	−0.08926	**0.993245**	−0.008249
PWN_M1OP	−0.007825	0.067204	**0.389266**
PWN_M2OP	0.006119	−0.026976	**0.716039**
PWN_Q1OP	−0.02271	0.00765	**0.44968**
PWN_Q2OP	−0.009645	−0.002484	**0.234895**
PWN_Y1OP	−0.000947	−0.008835	**0.279623**

Table 15: Summary of principal components analysis: Eigenvectors corresponding to the three largest eigenvalues (products explained in Section 4), PWN off-peak

A Appendix: Technical Derivations

Definition 1. *Let $K(n,t)$ be the wealth at time t of a trading strategy in which we start with unit wealth, and for each $i = 1, \ldots, n$, we invest all the wealth at time $\frac{i-1}{n}t$ to buy the electricity futures contract maturing at time $\frac{i}{n}t$, and hold that contract until maturity. We define K_t (the wealth of the* sliding MWh *trading strategy) to be*

$$K_t = \lim_{n\to\infty} K(n,t).$$

Proposition 2. *The wealth of the* sliding MWh *trading strategy in the one-factor market model (see Section 3) satisfies the following SDE:*

$$\frac{dK_t}{K_t} = \sigma d\widetilde{W}_t.$$

Proof. Let us fix a time t. It is easy to see that

$$\frac{K(n,t)}{K(n,0)} = \prod_{i=1}^{n} \frac{F(\frac{i}{n}t, \frac{i}{n}t)}{F(\frac{i-1}{n}t, \frac{i}{n}t)},$$

and taking logarithms on both sides:

$$\ln K(n,t) - \ln K(n,0) = \sum_{i=1}^{n} \left[\ln F\left(\frac{i}{n}t, \frac{i}{n}t\right) - \ln F\left(\frac{i-1}{n}t, \frac{i}{n}t\right)\right]. \quad \text{(A.1)}$$

By direct application of Itô's lemma, we obtain

$$\ln F\left(\frac{i}{n}t, \frac{i}{n}t\right) - \ln F\left(\frac{i-1}{n}t, \frac{i}{n}t\right) = -\frac{\sigma^2}{4\alpha}\left(1 - e^{-2\alpha t/n}\right) + \sigma \int_{\frac{i-1}{n}t}^{\frac{i}{n}t} e^{-\alpha(it/n - s)} d\widetilde{W}_s.$$

Substituting into (A.1) and rearranging gives

$$\ln K(n,t) - \ln K(n,0) = -\frac{\sigma^2}{4\alpha} n \left(1 - e^{-2\alpha t/n}\right) + \sigma \int_0^t h(n,s) d\widetilde{W}_s, \quad \text{(A.2)}$$

where for $s \in [\frac{i-1}{n}t, \frac{i}{n}t]$, $h(n,s) = e^{-\alpha it/n - s}$, with $i = 1, \ldots, n$. It is clear that

$$\lim_{n\to\infty} n(1 - e^{-2\alpha t/n}) = 2\alpha t. \quad \text{(A.3)}$$

Since $\lim_{n\to\infty} \int_0^t |h(n,s) - 1| ds = 0$, we have by elementary properties of the Itô integral (see for example Karatzas and Shreve (1997)) that

$$\lim_{n\to\infty} \int_0^t h(n,s) d\widetilde{W}_s = \widetilde{W}_t. \quad \text{(A.4)}$$

Taking the limit in (A.2), and substituting from (A.3) and (A.4), we get

$$\ln K_t - \ln K_0 = -\frac{\sigma^2}{2}t + \sigma\widetilde{W}_t.$$

Rewriting this in differential form, and further application of Itô's lemma completes the proof. □

Corollary 3. *In the three-factor market model, the wealth of the sliding MWh strategy satisfies*

$$\frac{dK_t}{K_t} = \sum_{i=1}^{3}\sigma_i d\widetilde{W}_t^i.$$

The proof is straightforward, and closely follows that above.

Proposition 4. *Given a series of observations* $\{Z_t\}_{t=1}^r$, $Z_t \in R^m$ *and matrices* $\{H_t\}_{t=1}^r$, $H_t \in R^{m\times n}$, *denote:*

$$X_t^* = \underset{X_t\in R^n}{\operatorname{argmin}}\sum_{t=1}^{r}(Z_t - H_tX_t)^2, \quad \xi_t^* = Z_t - H_tX_t^*, \quad R^* = \operatorname{cov}(\xi_t^*), \quad Q^* = \operatorname{cov}(X_t^*).$$

Now consider a state-space model of the form:

$$Z_t = H_tX_t + \xi_t, \quad X_t = d + \varepsilon_t,$$

where $X, d \in R^n$, $\varepsilon_t \sim \text{i.i.d.N}(0, Q^*)$, *and* $\xi_t \sim \text{i.i.d.N}(0, R^*)$. *Further, denote by* $\{\widehat{X}_t\}_{t=1}^r$ *the Kalman filter estimator of the state sequence* $\{X_t\}_{t=1}^r$ *given the observation sequence* $\{Z_t\}_{t=1}^r$. *Then,*

$$X_t^* = \widehat{X}_t; \quad t = 1, \dots, r.$$

Proof. The equivalence follows from the following set of equations:

$$\begin{aligned}
\widehat{X}_t &= \mathrm{E}[X_t|Z_1,\dots,Z_t] \\
&= \mathrm{E}[X_t|Z_t] \\
&= \operatorname{cov}(X_t, Z_t)\operatorname{var}(Z_t)^{-1}Z_t \\
&= \operatorname{cov}(X_t^*, Z_t)\operatorname{var}(Z_t)^{-1}Z_t \\
&= \operatorname{cov}((H_t'H_t)^{-1}H_t'Z_t, Z_t)\operatorname{var}(Z_t)^{-1}Z_t \\
&= (H_t'H_t)^{-1}H_t'\operatorname{var}(Z_t)\operatorname{var}(Z_t)^{-1}\mathrm{Z}_t \\
&= X_t^*,
\end{aligned}$$

where the second equality follows from the independence structure of the state-space model, the third is a standard result from multivariate regression theory, the fourth is implied by the choice of matrices Q^* and R^*, and the fifth and the last by the fact that X_t^* is a least-squares estimator. □