Optimal Structure of Air Transportation Services when Environmental Costs are taken into account

Etienne Billette de Villemeur
University of Toulouse (IDEI & GREMAQ)

Kévin Guittet*
Centre d’Étude de la Navigation Aérienne

May 2004
Very Preliminary and incomplete - Please do not cite

Abstract

The consequences of environmental taxation on the supply of air-transportation services by a profit-maximizing monopolist are examined. We consider both a passenger related tax and an aircraft related tax, highlight their respective impact on prices and frequency and derive their optimal combination. It is shown that there is a trade-off between the recovery of environmental damages and the distortions created by the tax. If tax can be negative, i.e. if the air transportation industry can be subsidized, it is possible to decentralize the optimal situation. We establish the optimal taxation system in a more realistic setting where tax are to be positive and the ATM costs have to be recouped. To conclude we question the opportunity and the ability of (direct) regulatory mechanisms to address these issues in substitution and/or in addition to (the indirect action of) a fuel tax.


Keywords: air transportation, environmental costs, airport charges, fuel tax.

*Corresponding author. E-mail: guittet@recherche.enac.fr.
1 Long summary

Air-transportation markets have known a huge growth over the last decades. There is an increasing concern that this positive traffic growth may conflict with environmental objectives in general and the Kyoto agreements in particular. In this paper, we study how air-transportation services are to be structured to minimize their environmental costs or conversely, how environmental objectives are to be embodied in the design of airport charges and fuel taxes as to minimize their negative impact on the profitability of the industry.

Our study is oriented to the European market, where, despite deregulation, many connections are still operated by a single operator. We thus examine the consequences of environmental taxation on the supply of air-transportation services in a setting where these services are supplied by a (profit-maximizing) monopolist.

Both the passenger related tax and the aircraft related tax are considered. We highlight their respective impact on prices and frequency and derive their optimal combination. It is shown that there is a trade-off between the recovery of environmental damages and the distortions created by the tax. As a result, environmental taxes should not be “too high”. In particular zero-tolerance for emissions is never optimal.

It is well known that a profit-maximizing monopolist produces below the socially optimal level. In other words, if no environmental damages were to be deplored, it would be optimal for government to subsidize the industry as to allow a greater number of travellers to benefit from air-transportation services. This optimal support (subsidy) decreases when the negative externalities of air-transportation are accounted for and can be negative (a tax) if environmental damages are “too high”. We show that, if taxes can take any value (positive or negative), it is always possible to decentralize the optimal situation. We then shift to a more realistic setting where tax are to be positive and the ATM costs have to be recouped. We exhibit in this context the optimal structure of airport charges.

To conclude we question the opportunity and the ability of airport charges to address environmental issues in substitution and/or in addition to fuel taxation. Fuel taxes impact indirectly (through operational costs) on both the frequency of flights and the ticket prices. Thus previous considerations are modified when this additional instrument is taken into account. We adopt the same structure for the analysis and examine successively the optimal combination of passenger-versus aircraft-related airport charges, the optimal level of emissions and the best way to recoup for ATM costs.
Introduction

Air-transportation markets have known a huge growth over the last decades. There is an increasing concern that this positive traffic growth may conflict with environmental objectives in general and the Kyoto agreements in particular. As air transport now stands for about 4% of the world’s total energy consumption, and given that airport pricing schemes and ATC operators scarcely take externalities into account, there is much to gain from deriving socially optimum taxes. Accordingly, our aim in this paper is to understand how air-transportation services are to be structured to minimize their environmental costs or conversely, how environmental objectives are to be embodied in the design of airport charges and fuel taxes as to minimize their negative impact on the profitability of the industry.

Our study is oriented to the European market, where, despite deregulation, many connections are still operated by a single operator. We thus examine the consequences of environmental taxation on the supply of air-transportation services in a setting where these services are supplied by a (profit-maximizing) monopolist. We however depart from generic works on the taxation of a monopoly (e.g. Barnet (1980)) by endogeneizing the frequency of flights, hence capturing a major specificity of the air-transportation market.

Environmental taxes in the field of air-transportation are considered by Nero and Black (1998). A single company is allowed to choose the price as well as the number of flights, but the main parameter under interest is the network structure. The environmental externality is not explicitly modelled, its effects being described by the concept of "footprints of pollution", but not fully captured³. Accordingly, the effect of taxes is studied on the price and frequency choices, the network structure and the profit of the firm, while welfare considerations are not performed. Carlsson (2002) build on this article and integrates explicitly the environmental externality in the expression of the welfare. He computes the optimal "frequency-related" tax in the monopoly case as well as in the duopoly case. Though these recent studies on environmental externalities have focussed on the specific impact of hubbing, we consider here a single leg and rule out competition to specifically address the question of the optimal combination of a "passenger-related" and a "frequency-related" tax. We believe that such schemes may allow to gain some insight into the implementation of Ramsey pricing schemes, as derived in the work of Dissler (1993), Zhang and Zhang (1997 and 2001), Carlsson (2003), Pels and Verhoef (2003) and others.

The model

General setting

Our study is oriented to the European market, where, despite deregulation, many connections are still operated by a single operator. In this paper, we thus consider single city-pair operated by a monopolist. The supply of air-

³However, it is true that estimating the marginal environmental costs is a hard task: part of the pollutants is emitted in altitude, and is thus not taken into account in studies related to urban pollution, though contributing to some extent to global warming and acid rains...
transportation services is characterised by the ticket price \( p \) and the flight frequency \( f \). It induces a demand\(^2\) of journey \( X(p,f) \).

As in Billette de Villemeur (2004), from which we borrow the notations, the firms has to bear fixed costs \( F \) and operational costs. The latter are directly related to the frequency of connections and the size of the aircrafts. A one-way flight with an aircraft of capacity \( K \) translates into operational costs \( C(K) \) on the route under scrutiny. Since operational costs are supposed to be proportional to the number of flights, their total amount is thus \( f \, C(K) \). Observe that, if the company adjust in the long run the capacity \( K \) of the aircraft to the total traffic observed \( X \), the relation \( K = X/f \) holds. As a result operational costs may rewrite \( f \, C(X/f) \).

We shall assume this is the case all along the paper. The framework may however easily be adapted to situation in which aircrafts are not used at full capacity.

Following the structural form of the operational costs of the airline, we assume that the environmental costs are roughly proportional to the number of flights and writes \( f \, E(K) \), where \( K \) is the capacity of the operated aircraft. Similarly the other costs, namely the costs for the airports and the air traffic control costs are denoted \( f \, G(K) \).

The airline is subject to tax payments. These taxes denoted by \( \tau_x \) and \( \tau_f \) bears on the number \( X \) of passengers and on the number of flights \( f \) respectively.

Since we essentially address a normative issue, namely the optimal structure of taxation, all coordination problems are washed away by assuming that a single regulator sets these taxes. When considering policy issues, it should taken into account that, in practice, part of these taxes are decided at the airport level. Recall that, on top of these taxes that bears directly on transportation services, part of the operational costs are due to fuel taxation which is not modelled explicitly here.

**First best allocation**

Before to address the tax issues and in order to obtain a reference point, we now consider the problem of a planner that aims at maximizing the social welfare. The latter writes

\[
W(X,f) = S(X,f) - fC(K) - fE(K) - fG(K),
\]  

(1)

where \( S(X,f) \) stands for the consumer aggregate (gross) surplus. Typically, it might be convenient to decompose \( S(X,f) \) as the difference between the gross utility \( U(X) \) and the time-costs \( X \, \nu \, T(f) \) of the \( X \) journeys, where \( \nu \) denotes the “value of time” and \( T(f) \) the travel time. However, unless usefull for the interpretation we will stack to the general form as to allow the comparison with other results from the litterature that are not compatible with this decomposition. Remark also that the emissions \( E \), although being part of social welfare, do

\[\frac{1}{(2f)}\]

This demand can be derived in many different ways. If \( U(X) \) denotes the gross utility derived by the representative agent from \( X \) journeys and \( \nu \) denotes its value of time, it writes

\[
X(p,f) = \arg \max_X \left\{ S(X) - \left( p + \frac{\nu}{2f} \right) X \right\},
\]

where \( 1/(2f) \) stands for the average waiting-time of the representative consumer. This representation however abstract from the fact that the utility \( U(X) \) derived from a journey is usually related to the value of time of the agent. Our model is however more general than that and we shall use these specification for illustrative purpose only.
not enter as an argument of the function $S(\ldots)$. This is so because we assume that, despite consumers value environmental quality, they do not take it into account in their travelling decisions.

Assuming quasi-concavity of the objective function, the optimality conditions of the problem are given by:

$$
\partial_X S(X, f) = C'(K) + E'(K) + G'(K),
$$

and

$$
\partial_f S(X, f) = \left( C(K) - KC'(K) \right) + \left( E(K) - KE'(K) \right) + \left( G(K) - KG'(K) \right).
$$

The demand for services $X(p, f)$ is directly related to the consumers’ net surplus. More precisely,

$$
X(p, f) = \arg \max \{ S(X, f) - pX \}.
$$

It follows that, at equilibrium, the price exactly equates the marginal benefit of a trip for the marginal consumer:

$$
p = \partial_X S(X, f).
$$

Consequently, equation (2) rewrites

$$
$$

In other words, the ticket price should equate its total marginal cost, namely the sum of the marginal operational cost, the marginal environmental costs and the marginal cost for the airport and the traffic control authority.

Observe that, although not explicit in the displayed equation (6), the frequency $f$ plays an important role in the determination of the equilibrium demand and price. To see that, consider the decomposition $S(X, f) = U(X) - \nu X T(f) \frac{\partial}{\partial f} \left[ \frac{S(X, f)}{X} \right]$. Equation (5) rewrites

$$
\tilde{p} = p + \nu T(f) = U'(X).
$$

In other words, for any pair $(p, f)$, the demand $X$ is such that the marginal utility of a journey is exactly equal to the generalised price $\tilde{p}$ bear by the users i.e. the sum of the ticket-price $p$ and the time-costs $\nu T(f)$ expressed in monetary units. More generally, an increase of $f$ can be shown to have a twofold impact on consumers. First, it increases their gross benefits as a higher frequency is nothing but a higher quality of services. Second, if this increase in frequency does not induce congestion hence further delays, it also decreases their travel time $T(f)$, reinforcing further the induced benefits.

The marginal impact of $f$ on the different costs is also twofold, with two opposite effects. On the one hand, it increases the costs since all of them are proportional to the number of flights. On the other hand, however, this first and obviously dominant effect is mitigated by the induced (long-term) decrease of the capacity $K = X/f$. It follows that the equation (3) can be rewritten as:

$$
\int f \frac{\partial}{\partial f} \left[ \frac{S(X, f)}{X} \right] = \left( \frac{C(K)}{K} - C'(K) \right) + \left( \frac{E(K)}{K} - E'(K) \right)
$$

$$
+ \left( \frac{G(K)}{K} - G'(K) \right).
$$
In other words, the marginal benefit of an increase in frequency as computed on a per-passenger basis, namely \( \partial f / [S(X, f)/X] \), is directly proportional to the difference between average and marginal costs, for all sources of costs.

The optimal allocation as characterised by equations (6) and (7) does not appear to be a realistic situation since it does not require the consumers to pay for all the costs induced by the transportation system. If there are increasing returns to scale in the various dimensions of the transportation system\(^3\), which appear to be a reasonable assumption, a marginal pricing scheme charges passengers below the average costs inflicted to society. The implementation of the first-best allocation builds necessarily on (possibly implicit) transfers.\(^4\) Increasing returns to scale in the environmental dimension means that, on a per-passenger basis, bigger aircrafts pollute less than (relatively) smaller aircrafts. A consequence is that \( E'(K) < E(K)/K \); Hence at the first-best passengers are not required to compensate for the whole environmental externality they generate.

Remark also that the implementation of the first-best allocation may rely on the existence of subsidies since the price may not be sufficient to cover the costs of the firm, the airport and the ATC services. Indeed, the overall profit of the industry is expressed as

\[
\pi_T = -f [(C(K) - KC'(K)) + (G(K) - KG'(K))] - F + E'(K). \quad (8)
\]

Interestingly enough, the higher the magnitude of the environmental externalities as measured by \( E'(K) \), the higher the price \( p \) hence the lower the necessary subsidies (if any) for the industry to financially break-even.

Finally, note also that, whenever necessary, these subsidies are not necessarily directed to the firm. If transportation services are charged according to the first-best rule (6), whether the price \( p \) will allow the firm to break-even depends on the magnitude of both environmental, airport and ATC costs. Again, the higher these marginal costs, the higher the prices hence the more likely the firm will recoup operational costs \( C(k) \) and fixed costs \( F \). Airport and ATC services would however run a deficit.

Previous considerations are made by considering the case of a social planner that is able to control the whole industry and get access to subsidies whenever it appears to be desirable. We now shift to a more realistic situation where \((i)\) the air transportation firm is a for profit company, \((ii)\) airport and ATC services are independent from the firm and \((iii)\) the industry as a whole may not be subsidized. Airport and ATC services are charged to the firm by the means of two fees, \( \tau_x \) that is a per-passenger tax and \( \tau_f \) that is a per-aircraft tax. We assume that Airport and ATC services are state owned bodies so that taxes are set as to maximize the social welfare.

**Implementing the first-best with a profit-maximising firm**

Before we address the second-best problem mentioned above, we uncover the behaviour of a profit-maximising firm in this environment. Given \( \tau_x \) and \( \tau_f \),

\(^3\)The operational, the environmental and the airport and control dimensions.

\(^4\)Transfers are implicit if the only costs that are not covered by the ticket are the environmental costs. Despite there are no monetary transfers, the whole society has to bear the negative externalities caused by the sole users of the air-transportation services. Transfers are explicit if the firm, the airports or the ATM services receive subsidies.
the profit of the firm writes

\[ \pi(p, f) = (p - \tau_x) X(p, f) - f (C(K) + \tau_f) - F, \]  

(9)

where \( K \) stands for the capacity of the operated aircraft that is assumed to adjust in the long term to write \( K = X/f \). Assuming the problem to be convex, the profit-maximising pair \((p, f)\) is fully characterised by the system of F.O.C:

\[ X(p, f) + \left[p - C'(K) - \tau_x\right] \frac{\partial X}{\partial p} = 0, \]  

(10)

\[ (p - C'(K) - \tau_x) \frac{\partial X}{\partial f} - \left(C(K) + \tau_f\right) + \frac{X}{f} C'(K) = 0. \]  

(11)

After some manipulations, the system can be rewritten as:

\[ \frac{p - C'(K) - \tau_x}{p} = \frac{1}{\epsilon_{xp}}, \]  

(12)

\[-X \left(\frac{\partial X}{\partial f} / \frac{\partial X}{\partial p}\right) = C(K) + \tau_f - KC'(K), \]  

(13)

where

\[ \epsilon_{xp} = \frac{p}{X(p, f)} \left(\frac{-\partial X}{\partial p}\right) \]

is the (absolute value of the) price-elasticity of demand.

Both equations deserve a few comments. Equation (12) comes out as the “standard” profit-maximising markup in presence of taxation. Given the frequency \( f \) and the consequent long-term capacity \( K = X/f \), the marginal cost for the firm of offering services to one additional passenger is \( C'(K) + \tau_x \). The per-passenger earning are thus \( p - C'(K) - \tau_x \) and the benefits of a price increase have to be balanced with the resulting loss of passengers. Interestingly enough, this rule is not altered by the possibly complex interaction of frequency and demand.

In order to interpret equation (13), it is useful to consider the implications of the consumer program as displayed in equation (4). By differentiating equation (5) with respect to \( p \) and to \( f \), and combining both expressions, one may indeed establish that:

\[- \left(\frac{\partial X}{\partial f} / \frac{\partial X}{\partial p}\right) = \frac{\partial}{\partial f} [\partial X S(X, f)] = \frac{\partial p}{\partial f}. \]

In other word, the ratio measures the marginal impact of frequency on the equilibrium price. As a result equation (13) establishes that the frequency is thus set in such a way that the marginal benefits of an increase in the number of flights exactly equates the resulting net cost increase, namely the total cost of operating an aircraft \( C(K) + \tau_f \) minus the downshift due to the capacity adjustment effect \( KC'(K) \). In order to make the link with the standard “value of time” concept, one can somewhat specify further the model to write the surplus \( S(X, f) \) as the difference between the gross utility \( U(X) \) and the time costs \( X \nu T(f) \). In this specific case equation (13) rewrites

\[ \nu T'(f) = \frac{1}{X} [C(K) + \tau_f - KC'(K)]. \]

\[ ^5 \text{Nero and Black [6] suggest this may even be achieved in the short term thanks to an "active and competitive rental service".} \]
which makes plain the just invoqued argument.

Of interest is the fact that the profit-maximizing goal of the firm does not a priori forbid the government to reach the first-best allocation exhibited in the previous section. This can be done by setting the taxes \( \tau_x \) and \( \tau_f \) at the respective level:

\[
\begin{align*}
\tau_x &= E'(K) + G'(K) - \frac{C'(K) + E'(K) + G'(K)}{\epsilon_{X_p}} \\
\tau_f &= (r - 1) \left[ C(K) - KC'(K) \right] + r \left[ E(K) - KE'(K) + G(K) - KG'(K) \right]
\end{align*}
\]

where

\[ r = \frac{\partial}{\partial f} \left[ \frac{\partial S(X, f)}{\partial X} \right] \frac{\partial}{\partial f} \left[ \frac{S(X, f)}{X} \right] \]

is the ratio of the marginal value of an increase in frequency for the marginal consumer over the marginal value of an increase in frequency for the average consumer.

Again, equations (14) and (15) merits some comments. Equation (14) highlights the existence of two origins of divergence between the price set by the firm and the price displayed in equation (6) that would implement the first-best allocation. First, the firm bears neither the environmental costs nor the airport and ATC costs. This explains the first two components of the tax that aim to induce the firm to internalize both cost sources. Second, since the firm is profit-maximising, it makes a positive markup while marginal pricing would be socially optimal. This explains the last term on the right handside of equation (14). Notice that the tax \( \tau_x \) can be negative or positive. In order to reduce environmental damages and if the firms is required to pay for the airport and ATC services, the firm should obviously be taxed. This is the first effect. However, since a monopoly makes use of its market power to produce less than the socially desired quantity, implementing the first-best requires the firm to be subsidized. This is the second effect, a classical result in optimal tax theory. The magnitude of the tax (or subsidy) will depend on the intensity of both effects. However, if \( \epsilon_{X_p} \geq 1 \), which has to be the case for the monopoly pricing formula (12) to make sense, it is clear that taking into account environmental externalities, as well as airport and ATC services tends to decrease the extend to which air-transportation firms should be otherwise subsidized.

In order to interpret equation (15), two points have to be kept in mind. First, as already stated by Spence (1975), there is a “divergence between private and social benefits”. Indeed firms’decisions are based upon marginal values whereas socially optimum decisions are based on the average one. This explains the introduction of the factor \( r \) in the formula. Second, as already stated above, the firm ignores the impact of its decisions on environment and the burden it imposes on airport and ATC facilities. As a result, although part of the repercussion of frequency on consumers’welfare is taken into account by the firm (because it impacts on its profits), it should be given the correct incentives to take these costs into account. A reasonable assumption on \( r \) is that this factor is smaller than one\(^6\). That is, the marginal consumer can be expected to be the one with the lowest value of

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\(^6\)This is indeed the case if a CES utility function is used (e.g. in [2] and [6]).
time, or at least with a value below the average one. By focusing on the marginal consumer, the firm under-estimates the value of frequency for travellers. In other words, the firm tends to over-estimate the costs of an increase in frequency. As a result, the tax $\tau_f$ should be smaller than the social cost of a marginal increase in frequency. Whether it should be positive or negative depends again on the relative magnitude of the costs. If transportations services were inducing no costs but the operational costs beared by the firms, the firm should be subsidized in order to contemplate the observation of the socially optimum frequencies. However, if environmental costs as well as airport and ATC costs are taken into account, the socially optimal level of frequency comes out lower and the amount of subsidies becomes lower up to the point were it becomes positive, hence a tax.

Second best

They are several reasons for not implementing the first best. First, it may be politically difficult to justify the subsidization of the airline. Hence, a possible problem to consider could be the setting of optimal levels of taxes under non-negativity constraints, either for both taxes, or globally. But the main reason is that the implementation of the optimal taxes, even in the "good" case of strong environmental externalities, may not allow the operator to cover its costs. The problem of the cost recovery is indeed the good one, as air navigation service providers (as well as many airports) are precisely constrained to balance their budget in european countries. Practically, air traffic control operators set their tariffs proportionally to a unit rate which is computed in order to balance their budget.

The second-best problem then consists in the maximization of

$$W(X, f) = S(X, f) - fC_t(K) - fE(K),$$

s.t. \begin{align*}
\text{(i)} & \quad \text{the airline maximises its profit,} \\
\text{(ii)} & \quad \text{the operator balances its budget.}
\end{align*}

Condition $\text{(i)}$ is precisely described by the first-order optimality conditions of the airline (10) and (11), while condition $\text{(ii)}$ writes

$$\tau_x X + \tau_f f \geq F_c + fC_c(K).$$

Notice that the budget balance constraint (17) is written in such a way that positive profit is allowed for the regulator. Indeed, it would be questionable to impose the equality constraint if the first best achieves the full recovery of the ATC costs. In this case, depollution operations (...) could be financed.

We may now express the problem faced by the air control operator. The Lagrangian writes as follows

$$\mathcal{L}(X, f, \tau_x, \tau_f, \lambda_1, \lambda_2, \beta) = W(X, f) + \lambda_1 X \left[ p + X \frac{\partial X p}{\partial X} - C_m'(K) - \tau_x \right] + \lambda_2 f \left[ X \frac{\partial f p}{\partial f} - (C_m(K) - KC_m'(K)) - \tau_f \right] + \beta \left[ \tau_x X + \tau_f f - F_c - fC_c(K) \right],$$

where the $\lambda_i$'s are the Lagrangian multipliers related to the airline profit maximization conditions, while $\beta$ is associated with the budget constraint of the
ANSP. Notice that constraints are expressed in such a way that Lagrangian multipliers are homogeneous to each other. The optimality equations with respect to the taxes give

\[ (-\lambda_1 + \beta)X = 0, \]
\[ (-\lambda_2 + \beta)f = 0 \]

Assuming that operating the line is profitable, and hence that \( X \neq 0 \) and \( f \neq 0 \), we get \( \lambda_1 = \lambda_2 = \beta \). The optimality equations in \( X \) and \( f \) then writes

\[
\epsilon + \beta(\epsilon - 3) \beta \left( \begin{array}{c}
\frac{\partial}{\partial X} \left[ X^2 \frac{\partial^2 p}{\partial X^2} + fX \frac{\partial^2 p}{\partial X \partial f} \right] = E'(K) + (1+\beta)C_t'(K), \\
(r + 2\beta)X \frac{\partial p}{\partial f} + \beta \left[ X^2 \frac{\partial^2 p}{\partial X \partial f} + fX \frac{\partial^2 p}{\partial f^2} \right] = (1+\beta) [C_t(K) - KC_t'(K)] \\
\end{array} \right)
\]

Further computations using the first order conditions yield the expression of the taxes variations around the first best taxes \( \tau_x^* \) and \( \tau_f^* \) (though these ones have to be evaluated for the second best quantities \( f \) and \( X \)). We get

\[
\frac{r + 2\beta}{\beta} (\tau_f - \tau_f^*) = \frac{1}{r} [ (r-2) \partial_2 (C_t(K) + E(K)) - r\partial_2 E(K) ] \\
- (X^2 \partial_{Xf}^2 p + fX \partial_{f}^2 p),
\]

\[
\frac{\epsilon + \beta(\epsilon - 3)}{\beta(\epsilon - 1)} (\tau_x - \tau_x^*) = \frac{1}{\epsilon} \left[ 3 \partial_1 (C_t(K) + E(K)) - \epsilon \partial_1 E(K) \right] \\
-\frac{1}{K} \left( \frac{\partial_2 (C_t(K) + E(K))}{r} + [\tau_f - \tau_f^*] \right) \\
- (X^2 \partial_{XX}^2 p + fX \partial_{fX}^2 p),
\]

where, by commodity, the following notations are adopted

\[
\left\{ \begin{array}{c}
\partial_1 C = C'(K), \\
\partial_2 C = C(K) - KC'(K).
\end{array} \right.
\]

Given the sign of the budget constraint, the Kuhn and Tucker optimality conditions ensure that \( \beta \) is non-negative. If \( \beta \) is null, then we recover the first best taxes, an obvious result since this condition reflects the fact that the budget constraint is satisfied at the social optimum. If \( \beta \) is strictly positive, then we observe that none of the taxes will in general maintain its first-best value. Taking apart the second derivatives of the price appearing in these equations, we observe that the "frequency-related" tax is likely (say for \( r \) small enough and keeping in mind that the tax \( \tau_f^* \) in (23) has to be computed on the second best values of \( (X, f) \)) to be smaller than the first best one.
References


